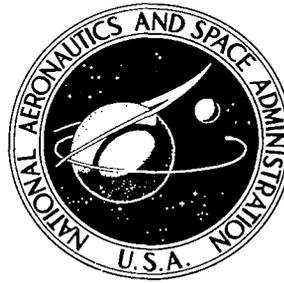


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**ZERO THERMAL EXPANSION COMPOSITES
OF HIGH STRENGTH AND STIFFNESS**

by Norris F. Dow and B. Walter Rosen

Prepared by
GENERAL ELECTRIC COMPANY
Philadelphia, Pa.
for Langley Research Center



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Prepared under Contract No. NAS 1-7659 by
GENERAL ELECTRIC COMPANY
Philadelphia, Pa.

for Langley Research Center

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

ABSTRACT

The results of a program to study the analysis and design of filamentary composites of zero coefficient of thermal expansion in one direction are reported. Guidelines are given for the selection of filamentary and binder materials and reinforcement configurations to achieve the desired thermal stability. Three-dimensional reinforcement configurations are shown to be substantially stronger, stiffer and more capable of achieving zero thermal expansion with a wider variety of material combinations than two-dimensional configurations.

FOREWORD

This document is the annual report on the program entitled "The Performance of a Study of Applications of Composite Materials to Aerospace Structural Design." The program was carried out by the Space Sciences Laboratory of General Electric for the National Aeronautics and Space Administration under Contract NAS1-7659, and was monitored by Mr. James P. Peterson of the NASA-Langley Structures Research Division.

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INTRODUCTION

For a number of engineering applications a material with dimensional stability — that is, high stiffness and a low coefficient of thermal expansion — is desirable. For aerospace applications low density and, in many cases, high strength are also desirable. In particular, design studies for orbiting optical systems (Refs. 1 and 2) have indicated the need for structural materials of high strength- and stiffness-to-weight ratios with essentially zero thermal expansion properties in at least one direction. Thus they may be employed in truss or space frameworks, for example, to achieve structures having overall dimensional stability.

Heretofore materials combining all of these desirable characteristics have not been available. Ceramic materials of low thermal expansivities have long been known (see, for example, Ref. 3), but in general they have exhibited low strengths or other unwanted properties. Metallic materials of high stiffness and low density (such as beryllium), while of higher strength than ceramics, do not achieve the desired thermal stability.

The advent of filamentary composites, especially those incorporating advanced materials like boron, has made accessible combinations of material properties not previously possible. Among these are thermal expansion coefficients which have been observed on the basis of preliminary calculations to vary substantially with reinforcement configuration, passing through a minimum (not too different from zero) for laminates with filaments at $\pm 30^\circ$ to the principal stiffness direction of the composite.

The present study was undertaken to explore in detail the possibilities of filamentary composites for meeting requirements of high strength and stiffness with zero thermal expansion in the principal stiffness direction, and with low weight. The prime objective was the establishment of guidelines for the generation of such materials. In this report these guidelines are given, together with descriptions of the analytical approaches used - and of the extensions to them developed as required as to carry through the development of the guidelines, example designs, and their evaluations.

A significant ancillary result, found when the evaluations were made, is that a transversely cubic* three-dimensional reinforcement configuration not only is more effective than a two-dimensional (laminate) configuration for controlling thermal expansions but also is so substantially stiffer and stronger that it is of importance for purely structural (non-thermal) applications. Accordingly, some discussion of the implications of this result is included.

*An orthotropic material in which 2 principal extensional moduli are equal is called transversely cubic.

PRELIMINARY SURVEYS OF THERMAL EXPANSIONS OF FILAMENTARY COMPOSITES

As a first step in the determination of the possibilities for control of thermal expansions by geometrical arrangements of the reinforcements in filamentary composites, a systematic survey was made of a number of combinations of filamentary and binder materials to establish the relative importance of the various factors influencing the expansivity. In this survey, first the properties of candidate materials themselves were considered, and then the effects of various combinations and configurations were analyzed, as described below.

Properties of Candidate Materials

Values of Young's Modulus, E , thermal expansion coefficient α , density, ρ , and, when available, Poisson's ratio, ν , for candidate filamentary and binder materials were compiled from published literature. These values are listed in Table 1 and plotted as values of E vs. α in Figure 1. Figure 1 points up the fact, not so evident from the numbers of Table 1, that materials in general fall into a band of values of stiffnesses and expansion coefficients such that the stiffnesses decrease as the expansion coefficients increase. The top of this band is characterized by materials such as tungsten carbide, boron, steel and zinc. The lower edge of the band includes Invar, glass, lead, and polyester. Fused quartz is one of the few materials falling outside the band thus defined, falling substantially below and to the left of the other materials.

Idealized Materials Used for Preliminary Evaluations

Evident in Figure 1 as indicated by the straight lines on the figure are families of materials having very nearly constant expansion coefficients with changing moduli, constant moduli and changing expansion coefficients, and moduli and expansion coefficients which change proportionally or inversely proportionally. With values slightly adjusted as indicated by the dashed lines between the plotted points, these materials provide the precisely consistently varying series given in Table 2, and these values were used as inputs for the systematic study of effects of property variations on expansion coefficients described in the subsequent section. The five families of "materials" selected in Table 2 provide the following four types of variations:

- (I) Changing expansion coefficient at constant modulus
- (II) Changing modulus at constant expansion coefficient
- (III) & (IV) Modulus varying inversely as the expansion coefficient
- (V) Modulus varying directly as the expansion coefficient.

Despite the idealizations, the values used for the material properties are close to those for real materials, i.e., feasible composites could reasonably be expected to be achievable with not too dissimilar properties from those calculated in this preliminary survey.

Preliminary Survey of Dependence of Expansion Coefficients of Composites on Properties of Constituents

Utilizing the LILAC II computer code (Ref. 4), elastic constants and thermal expansion coefficients were calculated for varying volume fractions of filaments and binders, for various angular

configurations of filamentary reinforcements, and for constituent properties selected from the five categories of Table 2 as follows:

- Case I. To study the influence of varying binder properties with filaments of maximum stiffness, calculations were made for composites with idealized "Tungsten Carbide" filaments in all of the other idealized materials of Table 2 as binders. Additionally for the "Polyester" material, the expansion coefficient was arbitrarily assigned values $\frac{1}{4}$, and 4 times the nominal value and Poisson's ratios of 0 and 0.5 were also used. For all other cases values of $\nu_f = \nu_b = 0.25$ were assumed. Results are given in Table 3.
- Case II. To study the influence of varying filament properties with binders of characteristic plastic-resin stiffnesses, calculations were made for composites with "Polyester" binders and all of the other idealized materials of Table 2 as filaments. Additionally, for the "Tungsten Carbide" material the expansion coefficient was arbitrarily assigned values $\frac{1}{4}$, and 4 times the nominal value. Results are given in Table 4.
- Case III. To study the influence of varying filament properties with binders of maximum stiffness, Case II was repeated for composites with the idealized "Tungsten Carbide" material as binder, with filaments of "Lead", "Babbitt", "Aluminum", "Steel", "Quartz", "Glass", "Zinc" and "Polyester" only. Results are given in Table 5.
- Case IV. To measure the relative importance of the binder and filament in the determination of the expansion coefficients of the composites, selected materials from the filaments of Case II were used as binders instead of filaments ("Lead", "Babbitt", "Aluminum", "Steel", "Quartz", "Glass", and "Zinc") and the binder material of Case II ("Polyester") was used as the filamentary material. Results are given in Table 6.
- Case V. The following additional combinations of materials were considered as being of especial interest:

Filaments		Binders
"Boron"	in	"Quartz"
"Invar"	in	"Quartz"
"Quartz"	in	"Invar"

Results are given in Table 7.

Results of Preliminary Survey

While the general trends revealed by the preliminary survey may be found in the numerical values listed in Tables 3-7, they tend to be somewhat obscured by the mass of data contained therein. Accordingly, illustrative examples have been selected and plotted as Figures 2 to 8 to emphasize the following results:

- (1) A ratio of filament to binder modulus of 10 is generally inadequate to produce the "scissoring" or "lazy tongs" action desired for zero thermal expansion composites. As shown in Figure 2 for a wide range of ratios of thermal expansion properties, there is no tendency toward the achievement of a thermal expansion coefficient even as low as that of the lesser of the filamentary or binder constituents, for any two-dimensional reinforcement configuration.
- (2) At a ratio of filament to binder modulus of 20, on the other hand (Fig. 3), some slight achievement of "scissoring" is evidenced by the minima to the thermal expansion curves in the vicinity of $\theta = \pm 25^\circ$. Values achieved, however, are still above those for the lesser of the expansions of the two constituents.
- (3) At even higher ratios of filament to binder modulus (25 and 50, see Fig. 4), the desired "scissoring" becomes pronounced. At the ratio of 50 ("WC" filaments in "Pb"), an expansion coefficient in the 1-direction less than that of the filamentary material (the constituent with the lower expansivity) is achieved for a wide range of angles θ (between about $\pm 15^\circ$ and $\pm 40^\circ$).
- (4) Negative values of thermal expansion in the 1-direction are achievable with stiff filaments in resin binders like the "Polyester" of Figure 5. In all cases these negative values are achieved for values of $\pm\theta$ in the range 25° to 40° , approximately (Fig. 6).
- (5) A high stiffness filament appears more effective for producing negative expansion coefficients than one of low thermal expansion (Fig. 6).
- (6) An increase in the thermal expansion coefficient of the binder can produce a substantial decrease in the α_1 of the composite laminate (Fig. 7). An increase in the binder Poisson's ratio can have the same effect (Fig. 8).
- (7) The only filamentary materials found in this preliminary survey to have the adequate combinations of high stiffness and low expansivity required to yield composites having negative thermal expansion coefficients were "Invar", "Tungsten", and "Tungsten Carbide". Disappointingly, "Boron" filaments did not quite achieve zero thermal expansions in any of the combinations or configurations considered. (Based on the results of (6) above, however, the prediction can be made that "Boron" filaments in a "Polyester" binder modified to give it an increased expansivity and Poisson's ratio could be made to give the desired zero expansion in the 1-direction.)
- (8) Of the combinations given in (7) which showed potentials for zero expansions, the "Tungsten" filaments appeared most representative of real, presently available materials (Invar and Tungsten Carbide are not known to be currently available as fine filaments). In order to define more precisely the characteristics achievable in filamentary composite of zero expansion, further calculations were carried out for the "Tungsten" filaments in "Polyester" binder at reinforcement angles determined by interpolation in Table 4 to yield near zero values of α_1 . The results of these calculations are given in Table 8.

METHODS OF ANALYSIS

The evaluation of effective properties of composite materials can be, and has been, studied with various methods of approach. Indeed, in the present study, more than one approach has been used in a complementary fashion. A brief explanation of the alternatives is therefore in order. The purpose of the definition of effective properties of heterogeneous materials is to provide relationships between average values of the state variables (stress, strain, temperature) which can be used in the same way that the relationships between local values of the state variables are used in the study of homogeneous media. Thus the logical definition of effective thermoelastic properties utilizes the form of the local stress-strain relations with state variables replaced by body averages of the local stress, strain and temperature, and with local properties replaced by effective properties. In a most general approach to this problem, thermoelastic variational procedures can be used to obtain bounds on each of the effective composite properties. This can be done for arbitrary phase geometry provided only that the geometry is statistically homogeneous. When the geometry is such that the type of elastic symmetry which exists is known, the problem is further simplified. For the studies of this type described herein, general stress-strain relations are utilized. Thus for the isothermal case we consider

$$\sigma_{ij} = C_{ijkl} \epsilon_{kl} \quad (1)$$

Here, results are obtained for materials of quite general elastic form. These can be reduced to simpler results for materials with a high degree of elastic symmetry.

In the other approaches utilized herein, the specific phase geometry (i.e., circular cylindrical fibers) is utilized. The results are thus applicable to certain special materials. In these treatments the general form of the elastic constant matrix is specialized to that of orthotropic (including transversely cubic and transversely isotropic) material. In this case the elastic constants are denoted as A_1, A_2, \dots etc. For these orthotropic materials, the principal results are obtained from the "3-D" analysis developed herein. In this analysis, the addition of fibers to a homogeneous matrix is treated in much the same fashion as the waffle-like addition of stiffeners to a flat plate. The effectiveness of these fibers is assessed by examination of the results obtained by the bounding procedure for the case of unidirectional fibers in a composite cylinder assemblage. A similar approach, which yields identical results is demonstrated in the failure criteria section of this report. Here it is assumed that the material is conceptually apportioned so that each fiber has associated with it a matrix volume such that the combination has the proportions appropriate to the overall volume fraction. The strain energy in each such volume element is approximated by the strain energy that would exist in a composite cylinder in which the same matrix volume was in the form of a cylindrical shell concentric with the fiber. The strain energy is evaluated for the case where the volume element is subjected to the boundary displacements that would exist at that cylinder surface if the composite were a homogeneous material subjected to the specified external boundary displacements. With the strain defined in this way, the unidirectional composite results can again be used to evaluate the strain energy, and hence the desired moduli.

Comparisons between the bounding method and the direct, but approximate, methods are available in those cases where the bounds are coincident. Thus the rigorous expression for the volume expansivity of an orthotropic composite provides a suitable means of confirmation of the present approach, as is illustrated subsequently.

EXTENSION OF 3-D ANALYSIS FOR CALCULATION OF THERMAL EXPANSION COEFFICIENTS

In view of the fact that the only two-dimensional combinations of materials found in the survey reported in the foregoing section to yield zero thermal expansions required the use of high density (Invar, W, or WC) filaments, consideration has been given to the use of three-dimensional reinforcement arrays. The basis for examining 3-D configurations was the assumption that control of thermal expansion in the one-direction should be achieved with reinforcing patterns which give equal transverse stiffnesses that is, equal "scissoring" action in both the two- and three-directions orthogonal to the one-direction. Methods for calculating the elastic constants of three-dimensionally reinforced filamentary composites were developed in Reference 5. In the following section these methods are extended for calculating thermal expansion coefficients.

Equations for Elastic Constants of Three-Dimensionally Reinforced Filamentary Composites

The equations for the elastic constants for three-dimensionally reinforced filamentary composites, derived in Reference 5 are reproduced in Table 9 in an equivalent but slightly modified form. The only changes are in the Equations for A4, A5, and A6, wherein the redundancies due to the equivalencies

$$\cos^2 \phi + \cos^2 \psi + \cos^2 \Omega = 1 \quad (2)$$

and

$$\frac{\beta_{\bullet T} (1 - \nu_f) - \beta'' (1 - 2\nu_f)}{\nu_f} = \beta_{\bullet G} \quad (3)$$

have been employed to simplify the form of the expressions, and the short-hand notation subscript $n = 1, 2, \text{ etc.}$, has been used to avoid repetitively writing out similar terms in full, where n identifies the filaments of modulus E_{fn} oriented at angles Φ_n, ψ_n , and Ω_n to the 1-, 2-, and 3- directions (see Fig. 9 for definition of Φ, ψ , and Ω).

As in Reference 5, the $A_1, A_2, \text{ etc.}$ represent the elastic constants sometimes termed "stiffnesses" in the stress-strain relations

$$\left. \begin{aligned} \sigma_1 &= A_1 \epsilon_1 + A_2 \epsilon_2 + A_3 \epsilon_3 \\ \sigma_2 &= A_2 \epsilon_1 + A_4 \epsilon_2 + A_5 \epsilon_3 \\ \sigma_3 &= A_3 \epsilon_1 + A_5 \epsilon_2 + A_6 \epsilon_3 \\ \tau_{12} &= A_7 \gamma_{12} \\ \tau_{23} &= A_8 \gamma_{23} \\ \tau_{13} &= A_9 \gamma_{13} \end{aligned} \right\} \quad (4)$$

The "stiffnesses" A_1 A_2 etc. are related to the more usual elastic moduli E_1 E_2 , etc., defined by the equations

$$\left. \begin{aligned} \epsilon_1 &= \frac{\sigma_1}{E_1} - \frac{\nu_{21}}{E_2} \sigma_2 - \frac{\nu_{31}}{E_3} \sigma_3 \\ \epsilon_2 &= -\frac{\nu_{12}}{E_1} \sigma_1 + \frac{\sigma_2}{E_2} - \frac{\nu_{32}}{E_3} \sigma_3 \\ \epsilon_3 &= -\frac{\nu_{13}}{E_1} \sigma_1 - \frac{\nu_{23}}{E_2} \sigma_2 + \frac{\sigma_3}{E_e} \end{aligned} \right\} \quad (5)$$

by the relations

$$\left. \begin{aligned} E_1 &= A_1 - \nu_{21} A_2 - \nu_{31} A_3 \\ E_2 &= A_4 - \nu_{12} A_2 - \nu_{32} A_5 \\ E_3 &= A_6 - \nu_{13} A_3 - \nu_{23} A_5 \end{aligned} \right\} \quad (6)$$

The "stiffnesses" are also related to the Poisson's ratios by the expressions

$$\begin{aligned} \nu_{21} &= \frac{A_2}{A_4} \left[\frac{1 - \frac{A_3 A_5}{A_2 A_6}}{\frac{A_5}{A_4 A_6}} \right] & \nu_{12} &= \frac{A_2}{A_1} \left[\frac{1 - \frac{A_3 A_5}{A_2 A_6}}{1 - \frac{A_3}{A_1 A_6}} \right] \\ \nu_{32} &= \frac{A_5}{A_6} \left[\frac{1 - \frac{A_2 A_3}{A_1 A_5}}{\frac{A_3}{A_1 A_6}} \right] & \nu_{23} &= \frac{A_5}{A_4} \left[\frac{1 - \frac{A_2 A_3}{A_1 A_5}}{1 - \frac{A_2}{A_1 A_4}} \right] \end{aligned} \quad (7)$$

$$\nu_{31} = \frac{A_3}{A_6} \left[\frac{1 - \frac{A_2 A_5}{A_3 A_4}}{1 - \frac{A_2}{A_4 A_6}} \right] \quad \nu_{13} = \frac{A_3}{A_1} \left[\frac{1 - \frac{A_2 A_5}{A_3 A_4}}{1 - \frac{A_2}{A_1 A_4}} \right] \quad (7)$$

cont'd.

and the usual reciprocal relations exists, i.e.

$$\nu_{12} = \nu_{21} \frac{E_2}{E_1} \quad (8)$$

and

$$\frac{\nu_{21}}{\nu_{12}} = \frac{\nu_{31} \nu_{23}}{\nu_{13} \nu_{32}} \quad (9)$$

Derivation of Equations for Thermal Expansion Coefficients

The derivation of the equations (Table 9) for the "stiffnesses" as described in Reference 5 involved the partial derivatives of the expression for the potential energy of the composite at a given temperature for imposed distortions $\epsilon_1, \epsilon_2, \epsilon_3, \gamma_{12}, \gamma_{23}$ and γ_{13} . If in addition to imposed distortions, a temperature rise ΔT is also imposed, the potential energy expression becomes that given as Table 10. Partial differentiation with respect to the strains, - as illustrated by equation (10) below, in which for simplicity only uni-directional filaments are considered, - yields equations identical to those for the elastic constants but with the addition in general of $-a(\Delta T)$ accompanying each extensional strain, thus

$$\begin{aligned}
\frac{\partial V}{\partial \epsilon_1} = & \frac{E_b(1-\nu_b)}{(1+\nu_b)(1-2\nu_b)} (\epsilon_1 - \alpha_b \Delta T) + \frac{2\nu_b E_b}{(1+\nu_b)(1-2\nu_b)} \left(\frac{\epsilon_2 + \epsilon_3}{2} - \alpha_b \Delta T \right) \\
& + \frac{E_f(1-\nu_f)}{(1+\nu_f)(1-2\nu_f)} \left(\beta_{\bullet_L} V_f \right) (\epsilon_1 - \alpha_f \Delta T) - \frac{E_b(1-\nu_b)}{(1+\nu_b)(1-2\nu_b)} \left(\beta_{\bullet_L} V_f \right) (\epsilon_1 - \alpha_b \Delta T) \\
& + \frac{2\nu_f E_f}{(1+\nu_f)(1-2\nu_f)} \left(\beta_{\bullet_{LT}} V_f \right) \left(\frac{\epsilon_2 + \epsilon_3}{2} - \alpha_f \Delta T \right) \\
& - \frac{2\nu_b E_b}{(1+\nu_b)(1-2\nu_b)} \left(\beta_{\bullet_{LT}} V_f \right) \left(\frac{\epsilon_2 + \epsilon_3}{2} - \alpha_b \Delta T \right)
\end{aligned} \tag{10}$$

Accordingly, when we collect the factors of ϵ_1 , ϵ_2 , ϵ_3 and ΔT in the differentiated equations, we find we now have expressions which may be written as follows:

$$\left. \begin{aligned}
\sigma_1 &= A_1 \epsilon_1 + A_2 \epsilon_2 + A_3 \epsilon_3 & - \left(A_{1\alpha} + A_{2\alpha} + A_{3\alpha} \right) \Delta T \\
\sigma_2 &= A_2 \epsilon_1 + A_4 \epsilon_2 + A_5 \epsilon_3 & - \left(A_{2\alpha} + A_{4\alpha} + A_{5\alpha} \right) \Delta T \\
\sigma_3 &= A_3 \epsilon_1 + A_5 \epsilon_2 + A_6 \epsilon_3 & - \left(A_{3\alpha} + A_{5\alpha} + A_{6\alpha} \right) \Delta T
\end{aligned} \right\} \tag{11}$$

where the portion of the equations (11) to the left of the vertical dashed line is identically as found before (Ref. 5) and the $A_{1\alpha}$, $A_{2\alpha}$, etc. can be written directly from the expression derived for A_1 , A_2 , etc. (Table 9) with a factor α_f added to each term containing E_f , and a factor α_b added to each term containing E_b . Thus (again for uni-directional filaments for simplicity)

$$\left. \begin{aligned}
A_{1\alpha} &= \frac{\alpha_b E_b (1-\nu_b)}{(1+\nu_b)(1-2\nu_b)} \left(1 - \beta_{\bullet_L} V_f \right) + \frac{\alpha_f E_f (1-\nu_f)}{(1+\nu_f)(1-2\nu_f)} \left(\beta_{\bullet_L} V_f \right) \\
A_{2\alpha} &= \frac{\alpha_b \nu_b E_b}{(1+\nu_b)(1-2\nu_b)} \left(1 - \beta_{\bullet_{LT}} V_f \right) + \frac{\alpha_f \nu_f E_f}{(1+\nu_f)(1-2\nu_f)} \left(\beta_{\bullet_{LT}} V_f \right)
\end{aligned} \right\} \tag{12}$$

etc.

Defining the expansion coefficients α_1 , α_2 , and α_3 in the 1-, 2-, and 3-directions as the ratio of the extensions to the rise in temperature, viz.

$$\left. \begin{aligned} \alpha_1 &= \left(\frac{\epsilon_1}{\Delta T} \right)_{\sigma_1 = \sigma_2 = \sigma_3 = 0} \\ \alpha_2 &= \left(\frac{\epsilon_2}{\Delta T} \right)_{\sigma_1 = \sigma_2 = \sigma_3 = 0} \\ \alpha_3 &= \left(\frac{\epsilon_3}{\Delta T} \right)_{\sigma_1 = \sigma_2 = \sigma_3 = 0} \end{aligned} \right\} \quad (13)$$

we note that by dividing equations (11) by ΔT we get

$$\left. \begin{aligned} A_1 \alpha_1 + A_2 \alpha_2 + A_3 \alpha_3 &= A_1 \alpha + A_2 \alpha + A_3 \alpha \\ A_2 \alpha_1 + A_4 \alpha_2 + A_5 \alpha_3 &= A_2 \alpha + A_4 \alpha + A_5 \alpha \\ A_3 \alpha_1 + A_5 \alpha_2 + A_6 \alpha_3 &= A_3 \alpha + A_5 \alpha + A_6 \alpha \end{aligned} \right\} \quad (14)$$

letting

$$\left. \begin{aligned} A_1 \alpha + A_2 \alpha + A_3 \alpha &= A_{\text{I}} \alpha \\ A_2 \alpha + A_4 \alpha + A_5 \alpha &= A_{\text{II}} \alpha \\ A_3 \alpha + A_5 \alpha + A_6 \alpha &= A_{\text{III}} \alpha \end{aligned} \right\} \quad (15)$$

and solving for α_1 , α_2 , α_3 in the same fashion that we previously solved equations (3) for ϵ_1 , ϵ_2 , and ϵ_3 , we find

$$\left. \begin{aligned} \alpha_1 &= \frac{A_{I\alpha} - \nu_{21} A_{II\alpha} - \nu_{31} A_{III\alpha}}{E_1} \\ \alpha_2 &= \frac{A_{II\alpha} - \nu_{12} A_{I\alpha} - \nu_{32} A_{III\alpha}}{E_2} \\ \alpha_3 &= \frac{A_{III\alpha} - \nu_{13} A_{I\alpha} - \nu_{23} A_{II\alpha}}{E_3} \end{aligned} \right\} \quad (16)$$

Evaluations of Transverse Effectiveness Coefficients

In Reference 5 equations are given for the evaluation of the filamentary transverse-effectiveness coefficients $\beta_{\bullet L}$, $\beta_{\bullet T}$, $\beta_{\bullet LT}$, $\beta_{\bullet G}$, $\beta_{\bullet'}$, and $\beta_{\bullet''}$ in terms of elastic constants for unidirectionally-reinforced composites. These equations employed the assumption that the transverse effectiveness coefficients $\beta_{\bullet L}$, $\beta_{\bullet LT}$, $\beta_{\bullet G}$, $\beta_{\bullet'}$ and $\beta_{\bullet''}$, used in the evaluation of the binder properties, were all equal to unity. This simplifying assumption, adequate for the calculation of elastic constants, is inadequate for the calculation of thermal expansion coefficients. Investigations undertaken to determine appropriate assumptions for the evaluations of the β 's involved systematic, extensive calculations employing comparisons between thermal expansion coefficients calculated by the LILAC II computer code and the 3-D analysis (for 2-D configurations), utilizing what appeared to be reasonable approximations for the interrelationships among the various β 's. The somewhat surprising result was obtained that, if the Poisson's ratios of filaments and binders are equal, the use of the assumption that

$$\left. \begin{aligned} \beta_{-\bullet L} &= \beta_{\bullet L} \\ \beta_{-\bullet LT} &= \beta_{\bullet LT} \\ \beta_{-\bullet G} &= \beta_{\bullet G} \\ \beta_{-\bullet T} &= \beta_{\bullet T} \\ \beta_{-\bullet'} &= \beta_{\bullet'} \\ \beta_{-\bullet''} &= \beta_{\bullet''} \end{aligned} \right\} \quad (17)$$

with the 3-D analysis causes the values of all elastic constants and thermal expansion coefficients for all two-dimensionally-reinforced configurations calculated by the LILAC II code and the 3-D equations of Table 9 to be identical. Further, the "average" expansion, derived from equation (33) also checks the results of the 3-D calculations. Accordingly the assumption of the equivalences of (17) was adopted for use with all 3-D calculations.

Criteria have not yet been found to establish relationships between filament-and binder-transverse effectivenesses which lead to exact consonance between the 3-D analysis and laminate analysis when the Poisson's ratios of filament and binder materials are not equal. The likelihood appears good (but it needs to be established) that a reasonable assumption leading to such consonance is

$$\beta_{-\bullet}^I = \beta_{\bullet}^I \quad (18)$$

and

$$\beta_{-\bullet}^{II} = \beta_{\bullet}^{II}$$

in view of the lack of Poisson expansions or contractions during simple shearing. Even if equation (18) is valid, however, an additional relationship is needed to permit the solution of the simultaneous equations for all the β 's in terms of known or calculated thermal expansion coefficients for uni-directional filament arrays. The basis for this additional relationship is not evident at present.

For want of a better criterion, therefore, the assumption represented by equation (17) was also employed herein for calculations when $\nu_b \neq \nu_f$. Checks of the 3-D analysis values resulting, against $\bar{\alpha}^*$ and LILAC II laminate calculations revealed only minor differences in thermal expansion coefficients (for examples, see Table 11). (Exact correspondence, of course, exists between the two approaches for all elastic constant values.) Accordingly results from the 3-D calculations for 3-D arrays are believed to lie well within the accuracies required for engineering analysis.

RIGOROUS BOUNDS FOR COMPOSITE THERMAL EXPANSION COEFFICIENTS

The evaluation of effective properties of composite materials has been carried out successfully in two different fashions. One is to obtain rigorous bounds on the particular property of interest that are applicable to a general geometry of the type considered. A second is to make approximations of the geometry to yield a configuration which is susceptible to exact analysis. The latter approach has been described in the section "Extension of 3-D Analysis for Expansion Coefficients" and the former approach is treated herein. Comparisons will be treated subsequently. Analyses have previously been conducted of the effective elastic moduli, thermal conductivity and specific heat of particulate and fibrous composites. The present work is directed toward the definition of the remaining property necessary for static thermoelastic analysis, namely, the effective thermal expansion coefficient of composites.

The approach used herein is to develop appropriate thermoelastic energy functions and variational principles to define bounds on such functions. When the energy functions are expressed in terms of macroscopic state variables and effective thermoelastic properties, the energy bounds can be used to bound the effective thermal expansion coefficients. Details of the analysis are presented in Ref. 6. The results are summarized herein.

Variational Principles

The concepts of classical thermodynamics applied to an isothermal elastic continuum can be shown to lead to the principles of Minimum Potential Energy and Minimum Complementary Energy (see Ref. 7). The similar problem for the case of temperature fields which depart from the reference state was studied in Ref. 6. It was shown that for a constant temperature process the Helmholtz free energy function, F , is a potential function for the stresses. Thus

$$\sigma_{ij} = \rho \left(\frac{\partial F}{\partial \epsilon_{ij}} \right)_T \quad (19)$$

ρ	density
σ_{ij}	stresses
ϵ_{ij}	strains
T	temperature

where the free energy is defined by:

$$F = E - TS \quad (20)$$

E = internal energy

S = entropy

Similarly it can be shown that for a constant temperature process, the Gibbs thermodynamic potential, G , is a complementary energy function in the sense that:

$$\epsilon_{ij} = - \rho \left(\frac{\partial G}{\partial \sigma_{ij}} \right)_T \quad (21)$$

where

$$G = F - \frac{1}{\rho} \sigma_{ij} \epsilon_{ij} \quad (22)$$

For a linear theory, the potential energy is given by:

$$W = \frac{1}{2} C_{ijkl} \epsilon_{ij} \epsilon_{kl} + \Gamma_{ij} \epsilon_{ij} T + B(T) \quad (23)$$

The minimum potential energy principle is valid for the present thermoelastic problem with this form of the potential energy and with $B(T)$ equal to any arbitrary function of temperature. The remaining constants are defined by the thermoelastic stress-strain relations:

$$\left. \begin{aligned} \sigma_{ij} &= C_{ijkl} \epsilon_{kl} + \Gamma_{ij} T \\ \sigma_{ij} &= C_{ijkl} (\epsilon_{kl} - \beta_{kl} T) \\ \epsilon_{ij} &= S_{ijkl} \sigma_{kl} + \beta_{ij} T \\ \epsilon_{ij} &= S_{ijkl} (\sigma_{kl} - \Gamma_{kl} T) \end{aligned} \right\} \quad (24)$$

C_{ijkl}	elastic moduli
S_{ijkl}	elastic compliances
β_{ij}	thermal expansion coefficients
$\Gamma_{ij} =$	$-C_{ijkl} \beta_{kl}$

For a linear theory, the complementary energy, W_c is given by:

$$W_c = \frac{1}{2} S_{ijkl} \sigma_{ij} \sigma_{kl} + \beta_{ij} \sigma_{ij} T + f(T) \quad (25)$$

The minimum complementary energy principle is valid for the present thermoelastic problem with this form of the complementary energy.

The logical definition of the effective thermal expansion coefficients follows from the use of body averages of the state variables in place of the state variables and from the use of effective properties in place of local properties in the stress-strain relations (24). Thus

$$\left. \begin{aligned} \bar{\sigma}_{ij} &= C_{ijkl}^* \bar{\epsilon}_{kl} + \Gamma_{ij}^* T \\ \bar{\epsilon}_{ij} &= S_{ijkl}^* \bar{\sigma}_{kl} + \beta_{ij}^* T \end{aligned} \right\} \quad (26)$$

With this definition of the effective expansion coefficients and with the availability of thermoelastic variational principles a desirable approach to the definition of effective properties in terms of phase properties is suggested. This is to express the energy functions in terms of the effective properties, postulate admissible stress or displacement fields to obtain bounds on the energy functions, and use these bounds to obtain bounds on the effective expansion coefficients.

The formulation of the free energy function in terms of average values and effective properties has been carried out only for a two phase material.

Isotropic Composites of Two Isotropic Phases

A statistically homogeneous and isotropic mixture of two isotropic phases is the first material considered. The result for the single effective expansion coefficients, α^* , of this composite is:

$$\alpha^* = \bar{\alpha} \left[\frac{\frac{1}{K^*} - \frac{1}{\bar{K}}}{\left(\frac{1}{K}\right) - \frac{1}{\bar{K}}} \right] + \frac{\bar{K} \alpha}{\bar{K}} \left[\frac{\left(\frac{1}{K}\right) - \frac{1}{K^*}}{\left(\frac{1}{K}\right) - \frac{1}{\bar{K}}} \right] \quad (27)$$

α = expansion coefficient

K = bulk modulus

overbar denotes volume average

asterisk denotes effective property

When the bulk modulus is known, equation (27) gives an exact result. Thus, for the composite spheres assemblage of Ref. 8 which is a material comprised of spheres of material 2 contained within concentric spherical shells of material 1, substitution of the exact solution for the effective bulk modulus into eq. (27) yields the expansion coefficient:

$$\alpha^* = \frac{\overline{K\alpha}}{\overline{K}} - \frac{3v_1v_2\phi(K_1 - K_2)^2}{\overline{K}(3K_1K_2 + 4\mu_1\overline{K})} \quad (28)$$

where

$$\phi = \frac{\alpha_2 - \alpha_1}{\frac{1}{K_1} - \frac{1}{K_2}}$$

If the bulk modulus is not known, but is bounded, then expression (27) generates bounds on the expansion coefficients. Since it is known (Ref. 9) that

$$\frac{1}{\overline{K}} \leq \frac{1}{K^*} \leq \overline{\left(\frac{1}{K}\right)} \quad (29)$$

it is clear from (27) that the effective expansion coefficient is a weighted average of the quantities $\overline{\alpha}$ and $\overline{K\alpha/K}$. Thus the actual value of α^* is bounded by these two quantities. Better bounds are obtained by using the arbitrary phase geometry best possible bounds of Ref. 10. These yield best possible bounds for the expansion coefficient of isotropic composites of arbitrary phase geometry.

Anisotropic Composites

The cases of practical interest for fibrous composites generally fall within the classification of an orthotropic effective material. For example, a composite composed of a homogeneous isotropic matrix containing a set of parallel circular cylindrical fibers forms a transversely isotropic material when the fiber cross-sections are randomly distributed over the transverse plane. For the orthotropic material there are three independent expansion coefficients, and the stress strain relations in compressed notation are given by:

$$\bar{\epsilon}_i = S_{ij}^* \bar{\sigma}_j + \alpha_i^* T \quad (30)$$

The procedure for evaluating the thermal expansion coefficients for this material is generally the same as that for the isotropic composite. The potential energy function for an anisotropic composite of two anisotropic constituents can be expressed as a function of mean values and effective properties only. By suitable choice of boundary conditions, a solution is obtained first for the mean orthotropic effective expansion coefficient, $\bar{\alpha}^*$. This quantity is simply:

$$\bar{\alpha}^* = \frac{1}{3} (\alpha_1^* + \alpha_2^* + \alpha_3^*) \quad (31)$$

where the α_i^* are the effective expansion coefficients in the principal directions. The mean effective expansion coefficient is clearly equal to one-third of the volumetric expansion. It is shown that for isotropic phases, the expression for $\bar{\alpha}^*$ is the same as that for α^* of the isotropic composite when the quantity ψ^* is substituted for the effective bulk modulus, K^* . Here

$$\frac{1}{\psi^*} = \sum_{i=1}^3 \sum_{j=1}^3 S_{ij}^* \quad (32)$$

Thus:

$$\bar{\alpha}^* = \bar{\alpha} \left[\frac{\frac{1}{\psi^*} - \frac{1}{\bar{K}}}{\left(\frac{1}{\bar{K}}\right) - \frac{1}{\bar{K}}} \right] + \frac{\bar{K} \bar{\alpha}}{\bar{K}} \left[\frac{\left(\frac{1}{\bar{K}}\right) - \frac{1}{\psi^*}}{\left(\frac{1}{\bar{K}}\right) - \frac{1}{\bar{K}}} \right] \quad (33)$$

It can be shown that $1/\psi^*$ is bounded by the same quantities as $1/K^*$ in (29) and hence it follows that $\bar{\alpha}^*$ is also bounded by $\bar{\alpha}$ and $\bar{K} \bar{\alpha}$.

With the arithmetic mean of the three effective expansion coefficients established, the potential energy function can be expressed in terms of a single effective expansion coefficient by suitable selection of boundary conditions. Bounds are then obtained for the individual values.

These are given by:

$$\alpha_{ib}^* = \frac{\overline{K\alpha}}{\overline{K}} - \frac{\varphi}{\overline{K}} + 3\varphi (S_{i1}^* + S_{i2}^* + S_{i3}^*)^{\pm} \quad (34)$$

$$6\varphi \left(\frac{1}{E_i^*} - \frac{1}{3\overline{\mu}} - \frac{1}{9\overline{K}} \right)^{1/2} \left(\frac{1}{\psi^*} - \frac{1}{\overline{K}} \right)^{1/2}$$

$$\varphi = \frac{\overline{\alpha} - \frac{\overline{K\alpha}}{\overline{K}}}{\frac{1}{\overline{K}} - \left(\frac{1}{\overline{K}} \right)} \quad (35)$$

These are the basic bounds for the expansion coefficients of an orthotropic composite. Appropriate solutions or bounds for the elastic moduli can be inserted into (34) to yield the results for any particular composite. Several special cases have been treated. First for equal phase Poisson ratios, $\nu_1 = \nu_2 = \nu$:

$$\alpha_{ib}^* = \frac{\overline{E\alpha}}{\overline{E}} - \frac{\varphi}{\overline{K}} + 3\varphi (S_{i1}^* + S_{i2}^* + S_{i3}^*)^{\pm} \quad (36)$$

$$6\varphi \left(\frac{1}{E_i^*} - \frac{1}{\overline{E}} \right)^{1/2} \left(\frac{1}{\psi^*} - \frac{1}{\overline{K}} \right)^{1/2}$$

For a unidirectional fibrous composite of arbitrary but isotropic transverse plane geometry and for equal phase Poisson ratios:

(37)

$$\alpha_1^* = \frac{\overline{E\alpha}}{\overline{E}}$$

$$\alpha_2^* = \alpha_3^* = \frac{\overline{E\alpha}}{\overline{E}} + \frac{3\varphi}{2} \left(\frac{1}{\overline{K}} - \frac{1}{\psi^*} \right) \quad (38)$$

Discussion

The analyses described to this point have developed bounds or expressions for the effective expansion coefficients of isotropic, transversely isotropic and orthotropic composites. The highlights of this work will be discussed in the Summary and Conclusions section. There are, however, certain limitations of the work which need further discussion. Principal among these is the fact that except for one relatively minor case, all the results apply only to two phase composites. The interest in multiphase material extends beyond the obvious desire to evaluate properties of composites of three or more phases, such as a matrix containing fibers of more than one type, or fibers of one material and particles of another. The other major interest is the desire to treat polycrystalline materials. For an anisotropic crystal, each change in orientation provides a different material with respect to the fixed reference axes of the composite. Thus a random mixture of anisotropic crystals may be viewed as a composite having an infinity of phases. The relationship between effective composite properties and constituent properties provides, in this case, a relationship between the properties of the isotropic polycrystalline material and the crystal properties.

All of the bounds derived herein for two phase composites can be expressed in terms of composite effective properties and volume averages of the phase properties. If the unknown multiphase results can also be expressed in terms of these average and effective quantities, then it follows that the present results are the desired results for multiphase materials. Intuitively, it seems reasonable to expect the multiphase results to be functions only of effective and average properties. Hence it may be that the present results are more general than indicated. However, the search for a proof of the hypothesis that the present results apply to multiphase materials has so far been fruitless.

A second question is associated with the limitation of the present treatment to boundary conditions which yield homogeneous solutions for the state variables when the material is isotropic whereas the more general practical problem is associated with the treatment of fields which are macroscopically variable. A rigorous answer to this question cannot be presented. However, it is important to emphasize that we are treating materials which are statistically homogeneous. That is, the volume average over any region which is large compared to a phase region, will be the same as the overall volume average. It is this concept which enables one to relate local values of the average state variables by effective thermoelastic constants or properties. In the treatment of macro-variable states, it is postulated that the material has local statistical homogeneity and that we may use the properties derived for homogeneous boundary conditions to relate the local average state variables.

It is intuitively apparent that this concept will break down when the macro-variables change their magnitude drastically over a dimension equal to a phase dimension. It has been suggested in Ref. (11) that we may treat the application of the effective properties to macro-variable problems as a "zeroth" approximation. One should recall that the theory of elasticity applied to polycrystalline materials assumes that average isotropic properties provide a valid representation of what is actually an inhomogeneous anisotropic material on the local or microscopic level. That this is a useful assumption is demonstrated by a successful history of analysis and experiment. That this assumption has limitations has been illustrated by the developments of multi-polar theories.

Another subject of discussion involves an alternate definition of the effective expansion coefficient; namely, a definition based on the energy expression. Thus the substitution of average values of the state variables for local values and of effective properties for local properties is made in the energy expression rather than in the stress-strain relation. For the zero temperature rise case the two approaches are identical. Thus, locally:

$$\sigma_{ij} = C_{ijkl} \epsilon_{kl} \quad (39)$$

$$W = \frac{1}{2} C_{ijkl} \epsilon_{ij} \epsilon_{kl} \quad (40)$$

These expressions suggest the following:

$$\bar{\sigma}_{ij} = C_{ijkl}^* \bar{\epsilon}_{kl} \quad (41)$$

$$\bar{W} = \frac{1}{2} C_{ijkl}^* \bar{\epsilon}_{ij} \bar{\epsilon}_{kl} \quad (42)$$

It can be shown that (42) follows from (41) and (40). Thus, in this case the energy definition and the average stress, average strain definition yield identical results for the effective elastic moduli. However, for the thermoelastic case, an energy definition following from the free energy expression would yield:

$$\bar{W} = \frac{1}{2} \bar{\epsilon}_{ij} (\bar{\sigma}_{ij} + \Gamma_{ij}^* T) + B^* \quad (43)$$

The average stress, average strain definition for this case yields quite a different energy expression. It appears that the two results do not yield the same definition of expansion coefficients for the thermoelastic case. The difference is illustrated by considering the special case of two phase

isotropic composites with zero surface tractions. For a homogeneous isotropic composite the free energy for this case would reduce from (43) to:

$$\bar{W} = -\frac{9}{2} K^* \alpha^{*2} T^2 + B \quad (44)$$

The energy expression would simplify to:

$$\bar{W} = \frac{9}{2} (\overline{K \alpha^2}) T^2 + \frac{9}{2} \varphi T^2 (\alpha^* - \bar{\alpha}) + \bar{B} \quad (45)$$

where φ is defined by equation (35).

For the two definitions to be equal, it would be necessary to define the local value of B as some scalar combination of the local moduli and expansion coefficients, such that when B* is set equal to the same combination of effective moduli and expansion coefficients equations (44) and (45) would be the same. Substitution for α^* can be made with equation (27), but efforts to show that (44) and (45) are the same have been unsuccessful. If it is not possible to use the effective properties in (44), or in (42) for the more general case, then these properties do not enable one to completely replace the actual heterogeneous material by the effective homogeneous material, and the thermoelastic treatment is not as rewarding as the elastic treatment. On the positive side is the fact that the more direct average stress, average strain treatment can utilize all the effective properties derived.

Summary

Effective coefficients of thermal expansion for certain types of composite materials have been derived. Despite the limitations discussed in the previous section, these results are considered useful for the evaluation of the thermoelastic response of practical composite materials. The general form of the results is a set of bounds on the composite expansion coefficient expressed in terms of the composite elastic moduli and the volume averages of the phase thermoelastic properties. The results are derived only for two phase composites. However, the form of the results does suggest a more general applicability. All of the results utilize phase properties at a given temperature, so that a composite having constituents with temperature dependent properties may readily be treated. It is evident that the existence of temperature dependent elastic moduli will result in temperature dependent composite expansion even if the phase expansion coefficients do not vary with temperature. In some cases, the effective elastic moduli will not be known. For certain of these cases there are theoretical bounds on the moduli available in the literature and these can be utilized. When the composite elastic moduli are known, the present results provide unique expressions for the effective expansion coefficient of an isotropic composite and for the effective volumetric expansion of an orthotropic composite. Bounds are presented for each of the principal expansion coefficients of transversely isotropic and orthotropic composites. For the case of equal phase Poisson ratios, the bounds on each of the expansion coefficients of a transversely isotropic composite are coincident.

The derivation of expansion coefficients was accomplished by the application of thermoelastic variational procedures developed in Ref. 6.

Consideration of the present results indicates the need for further study in several areas. First, the extension to multiphase materials should be confirmed by further theoretical treatment. Definition of the relationship between properties of isotropic polycrystalline materials and the crystal properties could then be accomplished. Comparison of the present results with experimental measurements would be most desirable. Also, the investigation of the energy definition of effective properties and the possibility of a better theory for macro-variable fields require theoretical treatment.

INVESTIGATION OF BORON/EPOXY COMPOSITES WITH THREE-DIMENSIONAL REINFORCEMENT

Using the equations developed for three-dimensional reinforcement (Table 9), calculations were made of the elastic constants for boron/epoxy composites of various volume fraction reinforcement and with various filament orientations. In all cases symmetrical reinforcement arrays were used, such that the filaments make equal angles (+ or - θ) with the 1-direction, i.e.

$$\left. \begin{aligned} \Phi &= \pm \theta \\ \psi &= \pm (90 - \theta) \text{ or } 90^\circ \\ \text{and } \Omega &= \pm (90 - \theta) \text{ or } 90^\circ. \end{aligned} \right\} \quad (46)$$

Thus with uniform filament spacings the composite has equal transverse stiffnesses.

Comparison calculations were made with the LILAC II computer program for similar two-dimensional boron/epoxy composites. Thus, in the 3-D notation,

$$\left. \begin{aligned} \Phi &= \pm \theta \\ \psi &= \pm (90 - \theta) \\ \Omega &= 90^\circ. \end{aligned} \right\} \quad (47)$$

For comparative purposes the total volume fractions of reinforcement in the 3-D configurations were made equal to those in the 2-D configurations – that is, the 3-D configuration can be considered made from the comparable 2-D configuration by taking $\frac{1}{2}$ of the filaments and rotating them about the 1-axis into a plane 90° from their original plane.

Results of the calculations are given in Tables 12 and 13, and plotted in Figure 10. As expected, the 3-D configuration does provide substantially more negative values of thermal expansions than the 2-D arrays, such that boron/epoxy combinations may be selected to give zero thermal expansion coefficients. For example, further calculation for $v_f = 0.4$ for $\theta = 34\frac{1}{2}^\circ$ yield the following values:

$$\begin{aligned} E_1 &= 6077 \text{ ksi} && (41.9 \frac{\text{GN}}{\text{m}^2}) \\ E_2 &= 1876 \text{ ksi} && (12.9 \frac{\text{GN}}{\text{m}^2}) \\ \alpha_1 &= 0 \\ \alpha_2 &= 34.6 \times 10^{-6} / ^\circ\text{F} && (62.3 \times 10^{-6} / ^\circ\text{K}) \end{aligned}$$

An unexpected result from these calculations was that the values of both E_1 and E_2 for the 3-D arrays were higher - in some cases substantially higher - than the comparable 2-D configurations. These enhanced stiffnesses have important implications for structural applications. These implications will be considered further in the evaluations of strengths and structural efficiencies in the following sections.

INVESTIGATION OF THE STRUCTURAL EFFICIENCIES OF ZERO- THERMAL-EXPANSION-COEFFICIENT COMPOSITES

For space vehicle structural applications requiring thermally stable configurations in general the loadings dictate that strength and stiffness properties be high at the same time that thermal expansions and material density be low. The combinations of high strength with low thermal expansion is not usually found in ceramic materials, for example (ref. 3), although low expansivities can indeed be achieved.

Alternate approaches to the achievement of thermally stable structures include, in addition to the filamentary composites considered here, bi-metallic configurations similar to the thermally-compensated pendulum designs known to clock makers for generations. Use of this type of approach for the applications under consideration leads to a structural element of the kind shown in Figure 11. For this element, obviously, a temperature rise expanding the titanium tube can be counteracted by the greater expansion of the aluminum tube to maintain the length L a constant.

The problem to be considered in this section, therefore, is the evaluation of various approaches to thermal structural stability, to determine whether the filamentary composites considered here offer potentially enhanced efficiency compared to the other approaches.

Comparative Evaluations-Tension

In order to evaluate low thermal expansion concepts for use as structures, two basic structural elements were considered, — a simple tension bar, and a thin-walled tube for compression. These elements were chosen because they can be used in space trusses and frameworks to utilize the low thermal expansion characteristics of the filamentary composite configurations along their lengths while accommodating lateral (diametral) expansions. Thus the measures of structural efficiency become merely the tensile strength/density ratio and the Young's modulus to density ratio in the lengthwise direction of the bar, and the tube-column efficiency for compression on the tube.

To establish a basis for comparison, the bimetallic element of titanium and aluminum alloys, sketched in Figure 11, was first considered for tensile loadings. For zero thermal expansion of the working section of length L, for

$$\left. \begin{aligned}
 \alpha_{7075-76} &= 11.7 \times 10^{-6} / ^\circ\text{F} \\
 &\quad (21.1 \times 10^{-6} / ^\circ\text{K}) \\
 \\
 \alpha_{\text{Ti}} &= 5.2 \times 10^{-6} / ^\circ\text{F} \\
 &\quad (9.36 \times 10^{-6} / ^\circ\text{K})
 \end{aligned} \right\} \quad (48)$$

the length of the titanium portion must be 1.8L, and that of the aluminum portion 0.8L. For equal strengths in the titanium and aluminum alloy portions, assuming that the tensile ultimate for the titanium is 1.75 times that of the aluminum alloy, the bimetallic bar weighs

2.43 times as much as an equal strength aluminum-alloy bar

or 2.68 times as much as a titanium bar of the same strength

(The assumed titanium alloy, having the higher ratio of tensile ultimate to density, is a lighter material than the aluminum alloy, in the ratio 2.68/2.43 when used alone, for this simple tension application).

If the bar is designed for stiffness rather than strength, the bimetallic construction is at a much greater disadvantage compared to either aluminum or titanium alloys. Carrying out the simple optimization problem (which yields the result that the aluminum and titanium portions of the bimetallic bar should be of about the same weight), leads to the result that the bimetallic construction has weights 7.5 and 8.1 times that of equal stiffness tensile bars of titanium and of aluminum, respectively.

Comparisons of the zero expansion composites with the bimetallic titanium-alloy aluminum-alloy zero expansion member for the tension application, utilizing the strength criteria developed in the following section of this report and the stiffness properties given in Tables 8 and 13, yield the values given in Table 14. As shown, the best "Tungsten"/"Polyester" composite designed by strength weighs 0.676 times as much as the corresponding bimetallic element. Likewise, the best "Tungsten"/"Polyester" composite designed by stiffness weighs only 0.526 times as much as the corresponding bimetallic bar. These values correspond to factors of 1.81 and 3.80 times the weights of titanium alloy by itself. Thus, while appreciably better than the bimetallic construction, the "Tungsten"/"Polyester" still requires that a heavy weight penalty be paid for thermal dimensional stability. This penalty, as previously anticipated, is primarily due to the high density of the tungsten.

The single three-dimensional boron/epoxy composite considered, on the other hand, would weigh only 0.367 times as much as the simple titanium tube designed for strength. The fact that it is still slightly heavier (1.12 times as heavy) as the non-zero-expansion titanium tube designed for stiffness is perhaps misleading; optimizing the proportions for axial stiffness to density ratio can undoubtedly improve this figure. In any event, the boron/epoxy of the arbitrarily selected proportions is only 0.149 times the weight of the best "Tungsten"/"Polyester" zero expansion laminate when stiffness is the criterion.

Comparative Evaluations – Compression

In compression, the efficiency of the bimetallic tube element can be evaluated in a manner similar to that used in Reference 5 for reinforced composite tube-columns. Thus the weight is simply

$$W = \rho_{Ti} A_{Ti} L_{Ti} + \rho_{Al} A_{Al} L_{Al} \quad (49)$$

where

- W = weight
- ρ = density
- A = cross-sectional area
- L = length

and the subscripts Ti and Al refer to the titanium and aluminum portions of the tube respectively.

Multiplying and dividing by P/L_{Ti}^2 etc., and using the length ratios previously derived for zero expansion gives

$$\frac{W}{L_{Ti}^3} = \rho_{Ti} \left[\frac{P/L_{Ti}^2}{P/A_{Ti}} \right] + \rho_{Al} \left[\frac{1}{2.25} \frac{P/L_{Ti}^2}{P/A_{Al}} \right] \quad (50)$$

Employing the assumption that the aluminum portion of the tube is always worked to 70 ksi (483 MN/m²), the compressive yield stress of the material (since it is the shorter, lower radius/thickness ratio portion), and letting

$$\frac{P}{A_{Ti}} = \sigma_{cr_{Ti}} = 0.3 E_{sec_{Ti}} (t/R)_{Ti} \quad (51)$$

gives

$$\frac{W}{L_{Ti}^3} = \rho_{Ti} \left[\frac{\frac{P}{L_{Ti}^2}}{0.3 E_{sec_{Ti}} (t/R)_{Ti}} \right] + \rho_{Al} \left[\frac{0.444}{70} \frac{P}{L_{Ti}^2} \right] \quad (52)$$

where

- t = wall thickness
- R = tube radius

E_{sec} = secant modulus

σ_{cr} = stress for local buckling of the tube wall

and

$$\frac{P}{L_{\text{Ti}}^2} = \frac{\left(0.3 E_{\text{sec}_{\text{Ti}}} \left\{t/R\right\}_{\text{Ti}}\right)^2}{\frac{\pi}{4} E_{\text{tan}_{\text{Ti}}} \left\{ \frac{2(R/t)_{\text{Ti}}^2 - 2(R/t)_{\text{Ti}} + 1}{2(R/t)_{\text{Ti}} - 1} \right\}} \quad (53)$$

as in Reference 6, with E_{tan} = tangent modulus.

Because the titanium portion of the tube must be 1.8 times the working (zero expansion) length, values of P/L^2 are 3.24 times those given by equation (53), and correspondingly values of W/L^3 are 5.832 times those given by equation (52). Values of P/L^2 and W/L^3 calculated by these relations are plotted in Figure 12 as the solid curve. Also plotted in Figure 12 is a curve for a simple titanium tube (long dashes), for the composite 2-D "Tungsten"/"Polyester" configurations (the short-dash curves) and 3-D boron/epoxy (dotted curves) configurations developed herein for zero thermal expansion. (Only the lightest and heaviest "Tungsten"/"Polyester" proportions are shown for clarity.

The calculations for the composite tubes were similar to those for the tubes of the other materials. Instead of $0.3 E_{\text{sec}}(t/R) = \sigma_{\text{cr}}$, the expression $\frac{1}{2}\sqrt{[E_1 E_2 / 3(1 - \nu_{21} \nu_{12})]} [t/R] = \sigma_{\text{cr}}$, was employed in the equations, after a survey showed that for all composites considered the shear stiffnesses were so great that the foregoing expression represented properly (see ref. 13) elastic buckling of the tube walls. For proportions such that the tube was strength limited, the stress values, σ_{cr} , found in Figure 13 were employed in place of the buckling stresses, σ_{cr} . The point of separation between these two stress cases are evident as the corners between the portions of the curves of differing slope on Figure 12. Thus the equations used become

$$\frac{W}{L^3} = \rho \left[\frac{\frac{P}{L^2}}{\left(\frac{1}{2}\right) \sqrt{\left(\frac{E_1 E_2}{3(1 - \nu_{21} \nu_{12})}\right) \left(\frac{t}{R}\right)}} \right] \quad (54)$$

or

or

$$\frac{W}{L^3} = \rho \left[\frac{P/L^2}{\sigma_{cu}} \right] \quad (55)$$

and

$$\frac{P}{L^2} = \frac{\frac{1}{4} \left[\frac{E_1 E_2}{3(1-\nu_{21} \nu_{12})} \right] \left(\frac{t}{R} \right)^2}{\frac{\pi}{4} E_1 \left[\frac{2 \left(\frac{R}{t} \right)^2 - 2 \left(\frac{R}{t} \right) + 1}{2 \left(\frac{R}{t} \right) - 1} \right]} \quad (56)$$

or

$$\frac{P}{L^2} = \frac{\sigma_{cu}^2}{\frac{\pi}{4} E_1 \frac{2 \left(\frac{R}{t} \right)^2 - 2 \left(\frac{R}{t} \right) + 1}{2 \left(\frac{R}{t} \right) - 1}} \quad (57)$$

The bimetallic tube/column is shown in Figure 12 to weigh approximately three times as much as a titanium tube at the same value of the loading index. The best 2-D "Tungsten"/"Polyester" tubes are almost identical in weight to the bimetallic titanium-aluminum tubes. The 3-D boron/epoxy, on the other hand is even slightly lighter than the simple, non-zero-expansion titanium tube.

Failure Criteria

The failure criterion for fibrous laminates treated in the structural efficiency study is the maximum stress criterion described in Ref. 5. Briefly, the failure condition is determined by comparison of each of the existing stresses in the principal elastic directions of a given lamina with the strength of that lamina when subjected only to the respective stress component. The laminate yield stress is defined as the average applied stress at which the first lamina stress component equals the defined strength level for that stress in that lamina. Ultimate strength is defined by utilizing a netting analysis. This failure criterion is discussed in detail in Ref. 5. For the materials treated in this study, the Young's modulus computed by netting analysis is zero. Hence, the ultimate strength and the yield strength coincide.

The laminate configurations considered herein consist of two sets of fibers making equal and opposite angles with the laminate reference axis. The strength criteria for this material postulate failure for any of the following conditions: extensional stress in the fiber direction equal to the uniaxial tensile or compressive strength of a unidirectional fibrous composite; average extensional stress in the direction normal to the fibers equal to the transverse strength of a unidirectional composite; or shear stress on the lamina principal axes equal to the in-plane shear strength of a unidirectional composite.

The failure criteria for the three-dimensional fibrous composites were developed by analogy to the approach for the laminates. These so-called 3-D composites had four sets of filaments in directions each making the same angle with the reference longitudinal axis and lying within one of two orthogonal planes parallel to the reference axis. The evaluation of ultimate strength was made by treating each set of filaments as if it were a lamina having the properties associated with its average volume fractions and embedded in the elastic composite material of known properties. Then stresses on principal axes are computed and compared with the maximum uniaxial allowable as in the laminate case. This procedure is described by the following analysis.

We consider the composite described above with the longitudinal axis denoted as the 1 axis, and with sets of filaments parallel to a pair of lines in each of the 1-2 and 1-3 planes. The angle between each set of filaments and the 1 axis is denoted θ . All fibers are of the same material and the local stress-strain relations, with respect to a set of orthogonal axes which includes the fiber axis and one of the material reference axes is given by:

$$\sigma_{ij} = C_{ijkl} \epsilon_{kl} \quad (58)$$

The elastic moduli, C_{ijkl} , are taken as those of a unidirectional composite of the reference fibers and binder having the same volume fractions as in the composite as a whole. This procedure cannot be justified on a rigorous basis, as the local properties associated with each of the four sets of fibers cannot be shown to be unaffected by the other sets. It is, however, put forth as a reasonable engineering approximation, and as will be shown it is identically equivalent to the approximations employed in the section entitled "Extension of 3-D Analysis for Calculation of Thermal Expansion Coefficients" (cf. equations (17)). The values used for the elastic moduli are those computed by the methods of Ref. 13 for the random array. (The upper bound is used for the transverse shear modulus).

The stress-strain relations of eq. (58) for any of the materials, denoted m , can be transformed to the laminate axes and expressed as:

$$\sigma'_{ij}(m) = C'_{ijkl}(m) \epsilon'_{kl}(m) \quad (59)$$

Here the primed quantities represent the results of the usual rotation of coordinates and they are functions of the respective unprimed quantities and the direction cosines. For example, the transformed moduli for the material oriented parallel to the line in the 1-2 plane which makes an angle θ with the positive 1 axis and an angle $90-\theta$ with the positive 2 axis are:

$$\left. \begin{aligned} C'_{1111}(a) = A'_1(a) &= A_1 \cos^4 \theta + A_4 \sin^4 \theta + (2A_2 + 4A_7) \sin^2 \theta \cos^2 \theta \\ C'_{1122}(a) = A'_2(a) &= A_2 (\cos^4 \theta + \sin^4 \theta) + (A_1 + A_4 - 4A_7) \sin^2 \theta \cos^2 \theta \\ C'_{1133}(a) = A'_3(a) &= A_2 \cos^2 \theta + A_5 \sin^2 \theta \\ C'_{2222}(a) = A'_4(a) &= A_1 \sin^4 \theta + A_4 \cos^4 \theta + (2A_2 + 4A_7) \sin^2 \theta \cos^2 \theta \\ \text{and } C'_{2233}(A) = A'_5(a) &= A_2 \sin^2 \theta + A_5 \cos^2 \theta \end{aligned} \right\} (60)$$

$$C_{ijkl} = \begin{bmatrix} A_1 & A_2 & A_{12} & 0 & 0 & 0 & 0 & 0 & 0 \\ & A_4 & A_5 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & A_4 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & \frac{1}{2}(A_4 - A_5) & 0 & 0 & \frac{1}{2}(A_4 - A_5) & 0 & 0 & 0 \\ & & & A_7 & 0 & 0 & A_7 & 0 & 0 \\ & & & & A_7 & 0 & 0 & A_7 & 0 \\ & & & & & \frac{1}{2}(A_4 - A_5) & 0 & 0 & 0 \\ & & & & & & A_7 & 0 & 0 \\ & & & & & & & A_7 & 0 \end{bmatrix} \quad (61)$$

We consider the material subjected to a prescribed set of constant boundary displacements. Since a representative volume element is the same at any point within the material, the average strains

resulting from these displacements are spacewise constant. Thus in any of the four materials which comprise this total composite, we may write from (59)

$$\sigma'_{ij}{}^{(m)} = C'_{ijkl}{}^{(m)} \epsilon'_{kl} \quad (62)$$

where

m denotes any of the four inclined materials

ϵ'_k are the constant strains for all values of m

The average stresses, σ'_{ij} , are defined for this case by:

$$\sigma'_{ij} = \frac{1}{4} \sum_{m=a}^d \sigma'_{ij}{}^{(m)} \quad (63)$$

Substituting (62) into (63):

$$\sigma'_{ij} = \frac{1}{4} \left[\sum_{m=a}^d C'_{ijkl}{}^{(m)} \right] \epsilon'_{kl} \quad (64)$$

The average stress, average strain relations (64) define a set of effective elastic moduli, C_{ijkl}^* as follows:

$$C_{ijkl}^* = \frac{1}{4} \sum_{m=a}^d C'_{ijkl}{}^{(m)} \quad (65)$$

The engineering moduli are defined in terms of these moduli. Thus, the effective longitudinal Young's modulus, E_1 is given by:

$$E_1 = A_1^* - 2\nu_{21} A_2^* \quad (66)$$

The effective longitudinal Poisson's ratio is defined by:

$$\nu_{21} = \frac{A_2^*}{A_4^* + A_5^*} \quad (67)$$

The correspondence between equations (6) and (7) and (66) and (67) is evident. With the effective properties known, the failure stresses can be determined in the fashion described earlier. This is treated here in detail for the case of a uniaxial applied stress, σ_0 , in the direction of the 1 axis. For this case:

$$\left. \begin{aligned} \sigma'_{11} &= \sigma_0 \\ \sigma'_{ij} &= 0 \quad \text{except for } i=j=1 \end{aligned} \right\} \quad (68)$$

For this case eqs. (65) yield:

$$\left. \begin{aligned} \epsilon'_{11} &= \frac{\sigma_0}{E_1} \\ \epsilon'_{22} &= \epsilon'_{33} = -\nu_{21} \frac{\sigma_0}{E_1} \\ \epsilon'_{12} &= \epsilon'_{23} = \epsilon'_{31} = 0 \end{aligned} \right\} \quad (69)$$

The strains (69) may be viewed as the constant strains in the materials associated with each of the four fiber directions. These strains can then be transformed to the respective principal axes for each fiber set. For example, treating the fiber set described previously, and rotating the strain components (69):

$$\left. \begin{aligned} \epsilon_{11}^{(a)} &= \frac{\sigma_0}{E_1} (\cos^2 \theta - \nu_{21} \sin^2 \theta) \\ \epsilon_{22}^{(a)} &= \frac{\sigma_0}{E_1} (\sin^2 \theta - \nu_{21} \cos^2 \theta) \\ \epsilon_{33}^{(a)} &= -\nu_{21} \frac{\sigma_0}{E_1} \\ \epsilon_{12}^{(a)} &= -\frac{\sigma_0}{E_1} (1 + \nu_{21}) \sin \theta \cos \theta \\ \epsilon_{23}^{(a)} &= \epsilon_{31}^{(a)} = 0 \end{aligned} \right\} \quad (70)$$

$$\left. \begin{aligned}
\sigma_{11}^{(a)} &= \frac{\sigma_o}{E_1} \left\{ A_1 (\cos^2 \theta - \nu_{21} \sin^2 \theta) + A_2 \left[\sin^2 \theta - \nu_{21} (1 + \cos^2 \theta) \right] \right\} \\
\sigma_{22}^{(a)} &= \frac{\sigma_o}{E_1} \left\{ A_2 (\cos^2 \theta - \nu_{21} \sin^2 \theta) + A_4 (\sin^2 \theta - \nu_{21} \cos^2 \theta) - \nu_{21} A_5 \right\} \\
\sigma_{33}^{(a)} &= \frac{\sigma_o}{E_1} \left\{ A_2 (\cos^2 \theta - \nu_{21} \sin^2 \theta) + A_5 (\sin^2 \theta - \nu_{21} \cos^2 \theta) - \nu_{21} A_4 \right\} \\
\sigma_{12}^{(a)} &= -\frac{\sigma_o}{E_1} \left\{ A_7 (1 + \nu_{21}) \sin \theta \cos \theta \right\} \\
\sigma_{23}^{(a)} &= \sigma_{31}^{(a)} = 0
\end{aligned} \right\} (71)$$

It follows, from these stresses, that the maximum shear stress in the transverse plane is:

$$\sigma_{23 \max} = \frac{1}{2} (A_4 - A_5) \left(\frac{\sigma_o}{E_1} \right) (1 + \nu_{21}) \sin^2 \theta \quad (72)$$

The elastic constants in (71) and (72) can be expressed in terms of engineering constants, thus

$$\begin{aligned}
A_1 &= E_1 + 4k_{23} \nu_{21}^2 \\
A_2 &= 2k_{23} \nu_{21} \\
A_4 &= k_{23} + G_{23} \\
A_5 &= k_{23} - G_{23} \\
A_7 &= G_{12}
\end{aligned} \quad (73)$$

and

$$k_{23} = \frac{A_4 + A_5}{2}$$

Now the stresses with respect to the fiber axes can be evaluated for a unit applied stress. With the maximum strength for each stress component known, the ratios of ultimate stress to existing stress are formed. This is done for the extensional stress, σ_{11} , and the shear stresses σ_{12} and σ_{23} . (In the two dimensional laminate σ_{22} is used in place of σ_{23} .) The lowest of these three ratios is the material strength. The strength for boron-epoxy composites of two and three dimensional construction having two and four fiber directions is treated as an illustration. The results for a fiber volume fraction of four tenths are shown in Fig. 13. The unidirectional strengths were taken as:

$$\left. \begin{aligned}
 \sigma_{11ult} &= \pm 300 \text{ ksi} \\
 \sigma_{12ult} &= \pm 10 \text{ ksi} \\
 \sigma_{22ult} &= \pm 20 \text{ ksi} \\
 \sigma_{23ult} &= \pm 10 \text{ ksi}
 \end{aligned} \right\} (74)$$

Curves for failure due to each of the three stress components (σ_{11} , σ_{12} , and σ_{22} for two dimensional laminates and σ_{11} , σ_{12} and σ_{23} for three dimensional materials) are shown separately. The material strength is the lower envelope to the three curves for each material. The striking strength improvements which result from the use of non-coplanar fiber arrays are evident. For example, for 30° composites a nearly four-fold strength improvement results. This is a consequence of the elimination of planes of shear weakness. Thus failure due to the shear stress component rather than the fiber extensional stress component is delayed from the 15° laminate to the 30° inclination for the 3-D composite. The effect of these results is indicated in the structural efficiency discussion.

RESULTS

Results of this investigation indicate that filamentary composite materials may be made with a zero coefficient of thermal expansion in one of their principal directions. The achievement of this zero expansion requires the use of material combinations and configurations which meet the following guidelines:

- (1) The filamentary material must combine high stiffness with a low coefficient of thermal expansion.
- (2) The binder material must combine a low stiffness with high values of thermal expansion coefficient and Poisson's ratio.
- (3) The reinforcement configuration utilizes filaments oriented at angles in the neighborhood of 30° to the direction of zero thermal expansion. A greater range of combinations of materials can be made to achieve zero thermal expansion if a three-dimensional reinforcement pattern, providing orthogonal symmetry about the zero expansion direction, is used rather than a two-dimensional (laminar type) reinforcement pattern.

Methods of analysis have been developed for the evaluation of the elastic constants, the thermal expansion coefficients, and the strengths of two and three-dimensionally reinforced filamentary composites. Strengths, stiffnesses, and densities evaluated for selected composites designed to give zero thermal expansion coefficients in one direction compare favorably with such structurally efficient materials as the better titanium alloys.

Three-dimensional reinforcement configurations were found to provide substantially enhanced strength properties, particularly for loadings for which shearing stresses inclined at angles to the filaments are critical. Stiffness properties of the three-dimensional reinforcement configurations are also generally superior to those for planar, two-dimensional reinforcements.

DISCUSSION

Factors affecting the dimensional stability of structures other than the simple one of coefficient of thermal expansion investigated in this study must be considered for end applications. Some of these factors will be discussed here in relation to the results obtained for filamentary composites. Additionally, implications regarding other characteristics than thermal stability will be reviewed in some areas - especially that of strength for three-dimensional reinforcement.

First, and this can be of major importance when end applications are considered, there is the fact that the transverse thermal expansion of many of the "zero-thermal-expansion" composites considered is large. The transverse expansion coefficient is not as large as the thermal expansion coefficient of the binder material alone, but it is not much less (see Table 3). To minimize the transverse expansion, the volume fraction filament should be maximized, and the angular reinforcement selected to give the desired zero expansion in one direction with the least transverse expansion. In general, this angle is the larger of the two angles θ (as in Figures 5 or 6, for example) at which the curve of α_l vs. θ passes through zero.

Second, because most, if not all materials change their expansion coefficients with temperature, zero expansivity can be achieved only over a limited range of temperature for any given combination. A degree of refinement not considered in this investigation but potentially available is the selection of filamentary and binder materials whose expansion coefficients change in complementary fashion with temperature such that the range of zero expansion is substantial, or at least adequate for the purpose intended.

Further exploration of low expansivity materials like quartz and Invar in three-dimensional configurations may reveal combinations more stable than those cited herein such as boron/epoxy. The use of metal matrices in place of plastic resins is attractive from the standpoint of long time stability, and zero expansion with metal matrices may be possible with optimized 3-D arrays as of quartz or Invar in a low-modulus alloy.

Perhaps the chief contribution of this study is the quantitative demonstration of the enhancement in shear strengths at angles to the filaments provided by 3-D reinforcement. The fact that the quantitative increases have been calculated to be so substantial should provide impetus to the development of such three-dimensionally reinforced composites.

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TABLE 1. Values of Properties of Candidate Filamentary and Binder Materials

Material	E, ksi (GN/m ²)	$\alpha \times 10^6, \frac{1}{^\circ\text{F}}$ ($\frac{1}{^\circ\text{K}}$)	ρ, pci (Mg/m ³)	ν
Teflon	90 (0.62)	55 (99)	0.078 (2.16)	
Polyester	450 (3.11)	50 (90)	.046 (1.27)	
Epoxy	500 (3.45)	25 (45)	.050 (1.39)	0.35
Lead	2000 (13.8)	16 (28.8)	.410 (11.36)	
Babbitt	4200 (29.0)	11 (20)	.352 (9.75)	
Tin	6200 (42.8)	12.5 (22.5)	.264 (7.31)	.33
Magnesium	6500 (44.9)	14 (25.2)	.063 (1.75)	.35
Cadmium	8000 (55.2)	16 (28.8)	.313 (8.67)	.29
Aluminum	10,300 (71)	10 (18)	.098 (2.71)	.33
Silver	10,300 (71)	11 (20)	.379 (10.50)	.39
Fused Quartz	10,400 (71.8)	0.3 (0.54)	.080 (2.22)	.16
E-Glass	10,500 (72.5)	2.8 (5.0)	.091 (2.52)	.20

TABLE 1 . (continued)

Material	E, ksi (GN/m ²)	$\alpha \times 10^6, / ^\circ\text{F}$ (/°K)	$\rho, \text{ pci}$ (Mg/m ³)	ν
S-Glass	12,400 (85.6)	2.2 (4.0)	.090 (2.49)	
Zirconium	13,700 (94.5)	3.2 (5.8)	.23 (6.37)	0.33
Zinc	14,000 (96.6)	22 (39.6)	.258 (7.15)	.25
Manganese Bronze	15,000 (103.5)	12 (21.6)	.300 (8.31)	.67
Titanium	15,500 (107)	4.6 (8.3)	0.163 (4.52)	.34
YM-31A Glass	15,900 (110)		.104 (2.88)	
Silicon	16,300 (112)	2.8 (5.0)	.084 (2.33)	.44
Paladium	16,300 (112)	6.6 (11.9)	.433 (11.99)	.39
Aluminum Bronze	17,000 (117)	8.9 (16.0)	.270 (7.48)	
Copper	17,000 (117)	9.1 (16.4)	.324 (8.97)	.35
Vanadium	19,000 (131)	4.6 (8.3)	.220 (6.09)	.36
Hafnium	20,000 (138)	3.3 (4.9)	.412 (11.41)	.30

TABLE 1. (continued)

Material	E, ksi (GN/m ²)	$\alpha \times 10^6, /^{\circ}\text{F}$ (/ ^o K)	ρ , pci (Mg/m ³)	ν
Platinum	21,300 (147)	6.6 (11.9)	.433 (11.99)	0.39
Invar	21,400 (148)	0.8 (1.44)	.289 (8.01)	
Aluminum Oxide (polycrystalline)	25,000 (173)	4.4 (7.9)	.114 (3.16)	
Tantalum	27,500 (190)	3.6 (6.5)	.600 (16.62)	.35
Lockalloy	28,000 (193)	9 (16.2)	.076 (2.11)	
Hastelloy B	28,500 (197)	5.5 (9.9)	.334 (9.25)	
Cobalt	30,000 (207)	7.6 (13.7)	.322 (8.92)	.30
Chromium	36,000 (248)	3.4 (6.1)	.260 (7.20)	
Beryllium	44,000 (304)	6.4 (11.5)	0.067 (1.86)	.05
Molybdenum	45,000 (311)	3 (5.4)	.369 (10.22)	.33
Beryllium Carbide	46,000 (317)	7.5 (13.5)	.088 (2.44)	
Tungsten	50,000 (345)	2.5 (4.5)	.697 (19.31)	.30
Boron	55,000 (380)	4.6 (8.3)	.091 (2.52)	.2

TABLE 1. (continued)

Material	E, ksi (GN/m ²)	$\alpha \times 10^6$ /°F (/°K)	ρ , pci (Mg/m ³)	ν
Boron Carbide	65,000 (449)	2.5 (4.5)	.091 (2.52)	
Silicon Carbide	70,000 (483)	2.7 (4.9)	.123 (3.41)	
Iridium	76,000 (524)	3.7 (6.7)	.813 (22.52)	0.26
Titanium Carbide	85,000 (587)	2.8 (5.0)	.173 (4.8)	
Tungsten Carbide	96,000 (662)	2.3 (4.1)	.540 (14.96)	.21

TABLE 2. - Properties of Idealized Materials Used for Study of
Influence of Constituent Properties on Expansion Coefficients

	E, ksi (GN/m ²)	$\alpha \times 10^6$, /°F (/°K)
I. Materials of same Young's modulus		
(1) "Quartz"	10,000 (69)	0.3 (0.54)
(2) "Glass"	10,000 (69)	2 (3.6)
(3) "Aluminum"	10,000 (69)	10 (18)
(4) "Zinc"	10,000 (69)	30 (54)
II. Materials of same expansion coefficient		
(1) "Lead"	2,000 (13.8)	10 (18)
(2) "Babbitt"	4,000 (27.6)	10 (18)
(3) "Aluminum"	10,000 (69)	10 (18)
(4) "Steel"	30,000 (207)	10 (18)
III. Characteristic "low modulus" materials		
(1) "Polyester"	500 (3.45)	40 (72)
(2) "Lead"	2,000 (13.8)	10 (18)
(3) "Glass"	10,000 (69)	2 (36)
(4) "Invar"	26,000 (179)	0.78 (1.4)

TABLE 2. - (continued)

	E, ksi (GN/m ²)	$\alpha \times 10^6$, /°F (/°K)
IV. Characteristic "high modulus" materials		
(1) "Zinc"	10,000 (69)	30 (54)
(2) "Steel"	30,000 (207)	10 (18)
(3) "Boron"	60,000 (414)	5 (9)
(4) "Tungsten Carbide"	100,000 (690)	3 (5.4)
V. Un-characteristic "low expansion" materials		
(1) "Quartz"	10,000 (69)	0.3 (0.54)
(2) "Invar"	26,000 (174)	.78 (1.4)
(3) "Tungsten"	50,000 (345)	1.5 (2.7)
(4) "Tungsten Carbide"	100,000 (690)	3 (5.4)

TABLE 3. Thermal Expansion Coefficients α_1 and α_2 for Composite Laminates of "Tungsten Carbide" Filaments in Various Idealized Binders (Arbitrary units. For units of /°F multiply by 10^{-6} . For units of /°K multiply by 1.8×10^{-6} .)

Binder Material	ν_f	$\nu_f = \nu_b = 0.25$ except as noted.											
		$\pm \theta$, deg.											
		0		20		25		30		35		45	
		α_1	α_2	α_1	α_2	α_1	α_2	α_1	α_2	α_1	α_2	α_1	α_2
"Polyester"	0.2	3.73	36.5	0.19	33.2	-1.43	30.4	-2.71	26.2	-2.72	20.2	5.15	5.15
	.4	3.28	26.1	0.77	23.7	-0.39	21.8	-1.31	18.8	-1.36	14.5	4.05	4.05
	.6	3.12	17.2	1.59	15.7	0.89	14.5	0.34	12.7	0.33	10.1	3.66	3.66
	.8	3.05	9.61	2.38	8.90	2.08	8.34	1.87	7.49	1.90	6.31	3.44	3.44
"Lead"	.2	3.52	9.25	3.07	8.66	2.90	8.19	2.83	7.51	2.97	6.58	4.33	4.33
	.4	3.20	7.35	2.85	6.91	2.71	6.56	2.63	6.06	2.70	5.36	3.68	3.68
	.6	3.09	5.69	2.88	5.41	2.80	5.19	2.76	4.88	2.81	4.45	3.42	3.42
	.8	3.03	4.25	2.96	4.12	2.93	4.02	2.93	3.88	2.98	3.69	3.25	3.25
"Glass"	.2	2.71	2.15	2.71	2.21	2.69	2.25	2.67	2.31	2.63	2.37	2.51	2.51
	.4	2.87	2.39	2.87	2.45	2.86	2.48	2.85	2.53	2.82	2.59	2.72	2.72
	.6	2.93	2.42	2.94	2.65	2.93	2.68	2.92	2.71	2.90	2.75	2.83	2.83
	.8	2.98	2.82	2.97	2.84	2.97	2.85	2.96	2.86	2.95	2.88	2.92	2.92
"Invar"	.2	1.87	1.20	1.81	1.28	1.78	1.32	1.74	1.37	1.69	1.43	1.56	1.56
	.4	2.38	1.68	2.33	1.76	2.30	1.81	2.25	1.87	2.20	1.93	2.07	2.07
	.6	2.67	2.15	2.63	2.21	2.61	2.25	2.58	2.29	2.54	2.34	2.44	2.44
	.8	2.86	2.59	2.84	2.63	2.83	2.64	2.81	2.66	2.79	2.69	2.74	2.74
"Aluminum"	.2	5.00	8.95	5.06	8.51	5.15	8.22	5.31	7.85	5.57	7.40	6.40	6.40
	.4	3.91	7.25	3.91	6.88	3.96	6.63	4.07	6.31	4.28	5.90	4.99	4.99
	.6	3.44	5.67	3.45	5.42	3.49	5.26	3.57	5.04	3.72	4.78	4.19	4.19
	.8	3.17	4.26	3.21	4.13	3.24	4.06	3.30	3.96	3.37	3.84	3.59	3.59

TABLE 3. (continued)

Binder Material	v_f	$\pm \theta$, deg.											
		0		20		25		30		35		45	
		α_1	α_2	α_1	α_2	α_1	α_2	α_1	α_2	α_1	α_2	α_1	α_2
"Quartz"	.2	2.23	0.71	2.20	0.88	2.17	0.99	2.11	1.13	2.01	1.30	1.69	1.69
	.4	2.65	1.36	2.65	1.50	2.63	1.60	2.59	1.72	2.51	1.88	2.23	2.23
	.6	2.83	1.97	2.83	2.07	2.81	2.13	2.78	2.22	2.72	2.31	2.54	2.54
	.8	2.93	2.51	2.92	2.56	2.91	2.59	2.89	2.63	2.86	2.67	2.77	2.77
"Zinc"	0.2	10.7	25.9	10.9	24.2	11.3	23.1	11.9	21.7	12.9	20.0	16.1	16.1
	.4	6.52	19.4	6.51	17.9	6.70	17.0	7.14	15.8	7.93	14.2	10.7	10.7
	.6	4.69	13.3	4.74	12.3	4.89	11.7	5.21	10.9	5.76	9.86	7.58	7.58
	.8	3.66	7.86	3.80	7.37	3.93	7.08	4.14	6.70	4.43	6.25	5.27	5.27
"Babbitt"	.2	3.97	9.16	3.72	8.60	3.67	8.20	3.72	7.63	3.95	6.89	5.14	5.14
	.4	3.40	7.32	3.17	6.90	3.10	6.58	3.10	6.14	3.25	5.54	4.15	4.15
	.6	3.18	5.68	3.05	5.41	3.02	5.21	3.03	4.93	3.13	4.56	3.70	3.70
	.8	3.07	4.25	3.04	4.12	3.04	4.03	3.07	3.90	3.13	3.75	3.39	3.39
"Steel"	.2	6.81	8.64	6.98	8.43	7.08	8.31	7.19	8.17	7.34	8.01	7.45	7.45
	.4	5.17	7.14	5.33	6.91	5.43	6.78	5.55	6.63	5.70	6.45	6.07	6.07
	.6	4.17	5.67	4.29	5.49	4.37	5.40	4.46	5.28	4.58	5.14	4.80	4.80
	.8	3.49	4.29	3.56	4.19	3.61	4.14	3.66	4.08	3.72	4.01	3.93	3.93
"Tungsten"	.2	2.00	1.81	1.98	1.83	1.97	1.84	1.95	1.86	1.94	1.87	1.91	1.91
	.4	2.36	2.12	2.33	2.15	2.32	2.16	2.30	2.18	2.28	2.20	2.22	2.22
	.6	2.63	2.42	2.60	2.45	2.59	2.46	2.58	2.48	2.56	2.49	2.52	2.52
	.8	2.83	2.72	2.82	2.73	2.81	2.74	2.81	2.75	2.80	2.76	2.78	2.78
"Boron"	.2	4.41	4.59	4.43	4.57	4.44	4.56	4.45	4.54	4.47	4.53	4.50	4.50
	.4	3.95	4.18	3.97	4.15	3.99	4.14	4.00	4.12	4.02	4.10	4.08	4.08
	.6	3.57	3.77	3.59	3.75	3.61	3.74	3.62	3.72	3.64	3.71	3.70	3.70
	.8	3.26	3.38	3.27	3.37	3.28	3.36	3.29	3.35	3.30	3.34	3.30	3.30

TABLE 3. (concluded)

Binder Material	v_f	$\pm \theta$, deg.											
		0		20		25		30		35		45	
		α_1	α_2	α_1	α_2	α_1	α_2	α_1	α_2	α_1	α_2	α_1	α_2
Modified Polyester	.2	3.14	9.33	2.47	8.71	2.16	8.19	1.92	7.39	1.92	7.25	6.82	6.82
	.4	3.05	7.37	2.58	6.92	2.36	6.55	2.18	5.98	2.17	5.18	5.04	5.04
	.6	3.02	5.69	2.73	5.41	2.60	5.18	2.50	4.83	2.49	4.64	4.56	4.56
	.8	3.01	4.25	2.88	4.12	2.83	4.01	2.79	3.85	2.79	3.83	3.76	3.76
Modified Polyester	.2	6.08	145	-8.90	131	-15.8	119	-21.2	101	-21.3	75.9	12.11	12.11
	.4	4.17	101	-6.46	90.9	-11.4	82.6	-15.3	69.9	-15.5	51.9	7.44	7.44
	.6	3.52	63.4	-2.97	57.0	-5.95	51.9	-8.29	44.1	-8.33	33.0	5.80	5.80
	.8	3.20	31.0	3.59	28.0	-0.90	25.7	-1.81	22.1	-1.66	17.0	4.88	4.88
Modified Polyester	0.2	3.73	27.7	1.30	23.9	0.32	21.4	-0.32	17.9	-0.14	13.6	4.53	4.53
	.4	3.28	18.9	1.68	16.3	1.03	14.6	0.61	12.3	0.72	9.47	3.68	3.68
	.6	3.12	12.3	2.20	10.7	1.84	9.69	1.61	8.33	1.68	6.70	3.38	3.38
	.8	3.05	7.13	2.66	6.42	2.52	5.96	2.43	5.36	2.48	4.65	3.23	3.23
Modified Polyester	0.2	3.73	47.0	-1.51	47.4	-4.57	46.7	-7.86	44.2	-9.64	37.8	7.31	7.31
	.4	3.28	36.1	-1.06	38.6	-3.99	39.9	-7.89	41.0	-11.7	39.7	7.38	7.38
	.6	3.12	25.1	0.15	27.6	-2.02	29.4	-5.23	31.8	-9.43	34.3	8.90	8.90
	.8	3.05	14.1	1.64	15.4	0.23	16.3	-0.83	17.6	-2.59	19.1	7.67	7.67

TABLE 4. - Thermal Expansion Coefficients α_1 and α_2 for Composite Laminates with "Polyester" Binder with Various Idealized Filamentary Materials (Arbitrary units. For units of /°F multiply by 10^{-6} . For units of /°K multiply by 1.8×10^{-6} .)

Filament Material	v_f	$v_f = v_b = 0.25$ $\pm \theta$, deg.											
		0		20		25		30		35		45	
		α_1	α_2	α_1	α_2	α_1	α_2	α_1	α_2	α_1	α_2	α_1	α_2
"Zinc"	0.2	31.7	38.7	31.4	38.0	31.4	37.4	31.5	36.7	31.9	35.7	33.5	33.5
	.4	30.7	36.2	30.4	35.6	30.4	35.1	30.4	34.5	30.7	33.7	31.9	31.9
	.6	30.3	33.8	30.2	33.4	30.2	33.2	30.2	32.8	30.4	32.3	31.2	31.2
	.8	30.1	31.8	30.1	31.6	30.1	31.5	30.2	31.3	30.3	31.1	30.6	30.6
"Steel"	.2	11.9	36.9	9.77	34.3	8.94	32.3	8.49	29.2	8.99	25.1	14.9	14.9
	.4	10.7	28.6	9.11	26.8	8.43	25.3	8.01	23.0	8.28	20.0	12.5	12.5
	.6	10.3	21.5	9.35	20.3	8.96	19.4	8.73	18.0	8.93	16.1	11.6	11.6
	.8	10.1	15.4	9.76	14.8	9.63	14.4	9.59	13.7	9.75	12.9	11.0	11.0
"Boron"	.2	6.13	36.6	3.06	33.5	1.71	31.0	0.72	27.1	0.91	21.7	8.23	8.23
	.4	5.43	26.8	3.20	24.6	2.20	22.8	1.45	20.0	1.52	16.1	6.59	6.59
	.6	5.19	18.5	3.84	17.0	3.24	15.9	2.80	14.2	2.87	11.8	6.00	6.00
	.8	5.07	11.3	4.50	10.6	4.27	10.1	4.12	9.28	4.20	8.21	5.65	5.65
"Lead"	.2	25.0	34.4	25.7	33.3	26.2	32.7	26.8	31.9	27.5	31.1	29.2	29.2
	.4	18.2	27.8	18.8	26.7	19.3	26.1	19.8	25.3	20.6	24.4	22.4	22.4
	.6	14.3	21.4	14.8	20.6	15.1	20.1	15.6	19.5	16.1	18.9	17.4	17.4
	.8	11.8	15.5	12.1	15.0	12.3	14.8	12.5	14.5	12.8	14.2	13.5	13.5
"Babbitt"	.2	20.0	35.2	20.5	33.5	20.9	32.4	21.7	31.1	22.7	29.4	25.8	25.8
	.4	14.7	28.1	14.9	26.6	15.3	25.7	15.8	24.4	16.7	22.9	19.5	19.5
	.6	12.3	21.5	12.5	20.4	12.7	19.8	13.2	18.9	13.8	17.9	15.6	15.6
	.8	10.9	15.4	11.1	14.9	11.3	14.6	11.5	14.2	11.9	13.7	12.7	12.7

TABLE 4. (continued)

Filament Material	v_f	$\pm\theta$, deg.											
		0		20		25		30		35		45	
		α_1	α_2	α_1	α_2	α_1	α_2	α_1	α_2	α_1	α_2	α_1	α_2
"Aluminum"	.2	15.0	36.2	14.3	33.9	14.3	32.3	14.6	30.1	15.7	27.2	20.5	20.5
	.4	12.1	28.5	11.3	26.7	11.2	25.4	11.3	23.6	12.0	21.2	15.7	15.7
	.6	11.0	21.5	10.5	20.3	10.5	19.5	10.6	18.4	11.1	16.9	13.5	13.5
	.8	10.4	15.4	10.3	14.8	10.4	14.4	10.5	13.9	10.8	13.3	11.9	11.9
"Quartz"	0.2	6.92	35.0	5.98	32.0	5.93	29.8	6.41	26.8	7.81	23.0	14.2	14.2
	.4	3.07	24.7	2.05	22.4	1.84	20.7	2.03	18.3	2.96	15.2	7.91	7.91
	.6	1.58	15.5	1.02	14.0	0.93	12.9	1.11	11.4	1.75	9.41	4.90	4.90
	.8	0.79	7.42	0.71	6.66	0.78	6.16	0.97	5.49	1.35	4.65	2.77	2.77
"Glass"	.2	8.33	35.2	7.44	32.3	7.38	30.2	7.85	27.4	9.19	23.7	15.3	15.3
	.4	4.65	25.4	3.68	23.1	3.47	21.5	3.66	19.2	4.55	16.2	9.28	9.28
	.6	3.23	16.6	2.69	15.1	2.60	14.0	2.78	12.6	3.39	10.7	6.40	6.40
	.8	2.47	8.81	2.39	8.09	2.46	7.61	2.64	6.96	3.00	6.17	4.37	4.37
"Invar"	.2	3.58	35.8	1.02	32.5	0.05	29.9	-0.41	26.0	0.34	20.7	8.01	8.01
	.4	1.88	25.1	-0.14	22.7	-0.95	20.7	-1.42	17.9	-1.00	13.9	4.48	4.48
	.6	1.28	15.8	0.07	14.3	-0.40	13.1	-0.65	11.3	-0.35	8.86	3.07	3.07
	.8	0.97	7.79	0.53	7.04	0.39	6.48	0.37	5.68	0.60	4.63	2.17	2.17
"Tungsten"	.2	2.98	36.2	-0.25	32.8	-1.65	30.1	-2.62	25.9	-2.31	20.0	5.66	5.66
	.4	2.07	25.5	-0.31	23.0	-1.36	21.1	-2.12	18.1	-1.99	13.8	3.57	3.57
	.6	1.75	16.3	0.31	14.8	-0.31	13.5	-0.76	11.7	-0.64	9.07	2.79	2.79
	.8	1.60	8.38	1.00	7.64	0.77	7.07	0.63	6.23	0.75	5.08	2.33	2.33

TABLE 4. - (concluded)

Filament Material	v_f	$\pm \theta$, deg.											
		0		20		25		30		35		45	
		α_1	α_2	α_1	α_2	α_1	α_2	α_1	α_2	α_1	α_2	α_1	α_2
Modified Tungsten Carbide	.2	1.52	36.3	-2.23	32.7	-3.95	29.9	-5.31	25.4	-5.32	19.0	3.03	3.03
	.4	1.04	25.2	-1.62	22.7	-2.85	20.7	-3.83	17.5	-3.88	13.0	1.86	1.86
	.6	0.88	15.8	-0.74	14.3	-1.49	13.0	-2.07	11.0	-2.08	8.25	1.45	1.45
$\alpha = \frac{1}{4} \alpha_o$.8	0.80	7.76	0.09	7.01	-0.22	6.42	-0.45	5.51	-0.41	4.26	1.22	1.22
Modified Tungsten Carbide	.2	12.5	37.3	9.88	34.8	8.64	32.8	7.68	29.6	7.67	25.0	13.6	13.6
	.4	12.2	29.5	10.3	27.7	9.43	26.2	8.74	23.9	8.70	20.7	12.8	12.8
	.6	12.1	22.8	10.9	21.6	10.4	20.7	9.99	19.3	9.98	17.3	12.5	12.5
$\alpha = 4\alpha_o$.8	12.0	17.0	11.5	16.5	11.3	16.0	11.1	15.4	11.2	14.5	12.3	12.3

TABLE 5. - Thermal Expansion Coefficients α_1 and α_2 for Composite Laminates with "Tungsten Carbide" Binder with Various Idealized Filamentary Materials. (Arbitrary units. For units of /°F. multiply by 10^{-6} . For units of /°K multiply by 1.8×10^{-6} .)

Filament Material	v_f	$v_f = v_b = 0.25$ $\pm \theta$, deg.											
		0		20		25		30		35		45	
		α_1	α_2	α_1	α_2	α_1	α_2	α_1	α_2	α_1	α_2	α_1	α_2
"Lead"	0.2	3.03	3.12	3.04	3.11	3.05	3.10	3.05	3.09	3.06	3.09	3.07	3.07
	.4	3.09	3.30	3.11	3.27	3.11	3.26	3.13	3.24	3.14	3.22	3.18	3.18
	.6	3.20	3.65	3.23	3.59	3.24	3.55	3.26	3.51	3.29	3.47	3.37	3.37
	.8	3.52	4.51	3.56	4.37	3.59	4.29	3.63	4.20	3.70	4.09	3.88	3.88
"Babbitt"	.2	3.07	3.22	3.08	3.20	3.09	3.19	3.10	3.18	3.11	3.17	3.14	3.14
	.4	3.18	3.56	3.21	3.51	3.22	3.48	3.25	3.45	3.27	3.42	3.34	3.34
	.6	3.40	4.15	3.44	4.05	3.47	4.00	3.51	3.93	3.56	3.86	3.70	3.70
	.8	3.97	5.45	4.04	5.25	4.09	5.14	4.16	5.00	4.26	4.85	4.53	4.53
"Aluminum"	.2	3.17	3.47	3.20	3.43	3.22	3.42	3.23	3.39	3.26	3.37	3.31	3.31
	.4	3.44	4.14	3.49	4.05	3.53	4.01	3.57	3.95	3.62	3.88	3.75	3.75
	.6	3.91	5.16	4.00	5.00	4.05	4.92	4.12	4.81	4.21	4.69	4.44	4.44
	.8	5.00	6.87	5.13	6.64	5.21	6.51	5.32	6.35	5.45	6.17	5.79	5.79
"Steel"	.2	3.49	3.96	3.54	3.90	3.56	3.87	3.60	3.84	3.63	3.80	3.71	3.71
	.4	4.17	5.09	4.26	4.98	4.31	4.92	4.37	4.85	4.44	4.78	4.61	4.61
	.6	5.17	6.46	5.29	6.31	5.37	6.22	5.45	6.13	5.55	6.02	5.78	5.78
	.8	6.82	8.10	6.95	7.95	7.02	7.87	7.10	7.77	7.20	7.66	7.43	7.43
"Quartz"	.2	2.93	2.82	2.92	2.83	2.92	2.84	2.91	2.85	2.90	2.86	2.88	2.88
	.4	2.83	2.56	2.81	2.59	2.80	2.61	2.78	2.63	2.76	2.66	2.71	2.71
	.6	2.65	2.17	2.62	2.23	2.59	2.26	2.57	2.30	2.53	2.35	2.44	2.44
	.8	2.23	1.51	2.18	1.60	2.15	1.65	2.11	1.71	2.05	1.78	1.92	1.92

TABLE 5. (concluded)

Filament Material	v_f	$\pm \theta$, deg.											
		0		20		25		30		35		45	
		α_1	α_2	α_1	α_2	α_1	α_2	α_1	α_2	α_1	α_2	α_1	α_2
"Glass"	.2	2.98	2.93	2.97	2.94	2.97	2.94	2.97	2.94	2.96	2.95	2.96	2.96
	.4	2.94	2.84	2.93	2.85	2.92	2.86	2.92	2.86	2.91	2.87	2.89	2.89
	.6	2.87	2.69	2.86	2.71	2.85	2.73	2.84	2.74	2.83	2.76	2.79	2.79
	.8	2.71	2.45	2.70	2.48	2.68	2.50	2.67	2.52	2.65	2.55	2.60	2.60
"Zinc"	.2	3.66	4.82	3.77	4.68	3.83	4.60	3.91	4.51	3.99	4.41	4.20	4.20
	.4	4.69	7.40	4.90	7.06	5.03	6.88	5.19	6.66	5.39	6.41	5.88	5.88
	.6	6.52	11.3	6.84	10.7	7.06	10.4	7.33	9.99	7.68	9.53	8.56	8.56
	.8	10.7	17.9	11.2	17.0	11.5	16.5	11.9	15.9	12.5	15.2	13.8	13.8
"Polyester"	.2	3.05	3.16	3.06	3.15	3.06	3.14	3.07	3.13	3.08	3.12	3.10	3.10
	.4	3.12	3.42	3.14	3.38	3.15	3.36	3.17	3.34	3.19	3.31	3.25	3.25
	.6	3.28	3.94	3.30	3.85	3.33	3.80	3.36	3.74	3.40	3.67	3.53	3.53
	.8	3.73	5.41	3.78	5.16	3.82	5.02	3.90	4.86	4.00	4.67	4.30	4.30

TABLE 6. - Thermal Expansion Coefficients α_1 and α_2 for Composite Laminates with "Polyester" Filaments and Various Idealized Binder Materials. (Arbitrary units. For units of /°F multiply by 10^{-6} . For units of /°K multiply by 1.8×10^{-6} .)

Binder Material	v_f	$v_f = v_b = 0.25$ $\pm \theta$, deg.											
		0		20		25		30					
		α_1	α_2	α_1	α_2	α_1	α_2	α_1	α_2	α_1	α_2	α_1	α_2
"Lead"	0.2	11.8	13.7	12.0	13.5	12.1	13.4	12.2	13.2	12.4	13.1	12.7	12.7
	.4	14.3	18.3	14.7	17.8	14.9	17.6	15.1	17.3	15.5	16.9	16.2	16.2
	.6	18.2	24.0	18.7	23.3	19.0	22.9	19.4	22.5	19.8	22.0	20.9	20.9
	.8	25.0	31.3	25.6	30.5	25.9	30.1	26.3	29.6	26.8	29.1	27.9	27.9
"Babbitt"	.2	10.9	12.4	11.1	12.2	11.1	12.1	11.2	12.0	11.3	11.9	11.6	11.6
	.4	12.3	15.7	12.6	15.3	12.7	15.0	13.0	14.8	13.2	14.5	13.8	13.8
	.6	14.7	20.5	15.1	19.7	15.4	19.4	15.8	18.9	16.2	18.4	17.2	17.2
	.8	20.0	28.0	20.6	27.0	21.0	26.4	21.4	25.8	22.0	25.0	23.5	23.5
"Aluminum"	.2	10.4	11.1	10.4	11.0	10.5	11.0	10.5	10.9	10.6	10.9	10.7	10.7
	.4	11.0	12.9	11.1	12.6	11.2	12.5	11.3	12.3	11.4	12.2	11.8	11.8
	.6	12.1	15.8	12.3	15.4	12.5	15.1	12.7	14.8	12.9	14.4	13.6	13.6
	.8	15.0	22.0	15.4	21.1	15.6	20.5	16.0	20.0	16.5	19.2	17.7	17.7
"Steel"	.2	10.1	10.4	10.1	10.4	10.2	10.4	10.2	10.3	10.2	10.3	10.3	10.3
	.4	10.3	11.1	10.4	11.0	10.4	10.9	10.5	10.9	10.5	10.8	10.6	10.6
	.6	10.7	12.4	10.8	12.1	10.9	12.0	10.9	11.9	11.1	11.7	11.4	11.4
	.8	11.9	15.6	12.0	15.1	12.1	14.8	12.3	14.4	12.5	14.0	13.2	13.2
"Quartz"	.2	0.79	1.81	0.88	1.69	0.93	1.62	1.00	1.54	1.08	1.45	1.26	1.26
	.4	1.58	4.12	1.76	3.80	1.87	3.62	2.02	3.41	2.20	3.17	2.66	2.66
	.6	3.07	8.03	3.35	7.39	3.54	7.03	3.81	6.60	4.16	6.11	5.07	5.07
	.8	6.92	16.2	7.39	14.9	7.74	14.2	8.22	13.4	8.85	12.5	10.5	10.5

TABLE 6. - (concluded)

Binder Material	v_f	$\pm \theta$, deg.											
		0		20		25		30		35		45	
		α_1	α_2	α_1	α_2	α_1	α_2	α_1	α_2	α_1	α_2	α_1	α_2
"Glass"	.2	2.47	3.45	2.56	3.33	2.61	3.26	2.67	3.18	2.74	3.10	2.92	2.92
	.4	3.23	5.65	3.40	5.35	3.50	5.18	3.65	4.97	3.82	4.74	4.26	4.26
	.6	4.65	9.40	4.92	8.79	5.11	8.44	5.36	8.03	5.70	7.56	6.57	6.57
	.8	8.33	17.2	8.79	16.0	9.12	15.3	9.58	14.5	10.2	13.7	11.8	11.8
"Zinc"	.2	30.1	30.4	30.1	30.3	30.2	30.3	30.2	30.3	30.2	30.3	30.2	30.2
	.4	30.3	31.0	30.4	30.9	30.4	30.8	30.4	30.8	30.5	30.7	30.6	30.6
	.6	30.7	31.9	30.8	31.8	30.8	31.7	30.9	31.6	31.0	31.5	31.2	31.2
	.8	31.7	34.0	31.8	33.7	31.9	33.5	32.0	33.3	32.2	33.1	32.6	32.6

TABLE 7. - Thermal Expansion Coefficients α_1 and α_2 for Composite Laminates of Selected Combinations of Filamentary and Binder Materials. (Arbitrary units. For units of $^{\circ}\text{F}$. multiply by 10^{-6} . For units of $^{\circ}\text{K}$. multiply by 1.8×10^{-6} .)

Filament/Binder	v_f	$v_f = v_b = 0.25$											
		$\pm \theta, \text{ deg.}$											
		0		20		25		30		35		45	
		α_1	α_2	α_1	α_2	α_1	α_2	α_1	α_2	α_1	α_2	α_1	α_2
"Boron"/"Quartz"	0.2	3.12	1.11	3.01	1.34	2.94	1.47	2.82	1.64	2.68	1.84	2.29	2.29
	.4	4.06	2.18	3.99	2.39	3.93	2.52	3.83	2.69	3.70	2.89	3.32	3.32
	.6	4.53	3.21	4.47	3.36	4.43	3.45	4.36	3.57	4.26	3.70	4.00	4.00
	.8	4.81	4.15	4.77	4.23	4.74	4.27	4.70	4.33	4.65	4.39	4.52	4.52
"Invar"/"Quartz"	.2	0.49	0.40	0.48	0.41	0.47	0.41	0.47	0.42	0.46	0.43	0.44	0.44
	.4	0.60	0.50	0.59	0.51	0.59	0.52	0.58	0.52	0.57	0.53	0.55	0.55
	.6	0.68	0.60	0.67	0.61	0.67	0.61	0.66	0.62	0.66	0.63	0.64	0.64
	.8	0.74	0.69	0.73	0.70	0.73	0.70	0.73	0.70	0.72	0.71	0.72	0.72
"Quartz"/"Invar"	.2	0.74	0.71	0.73	0.71	0.73	0.71	0.73	0.71	0.73	0.72	0.72	0.72
	.4	0.68	0.62	0.68	0.63	0.67	0.63	0.67	0.64	0.66	0.64	0.65	0.65
	.6	0.60	0.53	0.60	0.54	0.59	0.54	0.59	0.55	0.58	0.55	0.57	0.57
	.8	0.49	0.42	0.48	0.43	0.48	0.43	0.47	0.44	0.47	0.44	0.46	0.46

TABLE 8. Thermal Expansion and Stiffness Characteristics of "Tungsten"/"Polyester" Filamentary Composite Laminates Designed for Values of α_1 Approaching Zero.

v_f	$\pm \theta$, deg.	$\alpha_1 \times 10^6 / ^\circ\text{F}$ (/ $^\circ\text{K}$)	$\alpha_2 \times 10^6 / ^\circ\text{F}$ (/ $^\circ\text{K}$)	E_1 , ksi (GN/ m^2)	E_2 , ksi (GN/ m^2)
0.2	{ 25.7	0.028 (.050)	21.83 (39.29)	4559 (31.46)	740.5 (5.11)
	{ 37.1	0.013 (.023)	13.85 (24.93)	1740 (12.01)	773.3 (5.34)
0.3	{ 25.3	0.051 (.092)	19.22 (34.60)	6511 (44.93)	923.0 (6.37)
	{ 37.7	-0.033 (-.059)	11.55 (20.79)	2122 (14.64)	967.9 (6.68)
0.4	{ 27.1	0.021 (.038)	16.01 (28.82)	7234 (49.91)	1159 (8.00)
	{ 37.1	0.001 (.002)	10.57 (19.03)	2832 (19.54)	1202 (8.29)
0.5	{ 33.5	0.031 (.056)	11.16 (20.09)	5023 (34.66)	1487 (10.26)

TABLE 9. Generalized Equations for "Stiffnesses" of Three-Dimensionally Reinforced Filamentary Composite in Terms of the Transverse Effectiveness Coefficients (β).

$$\begin{aligned}
 A_1 = & \frac{E_b(1-\nu_b)}{1-\nu_b-2\nu_b} \left\{ 1-\beta_{\bullet L_n} \nu_{f_n} \cos^4 \phi_n - \beta_{\bullet T_n} \nu_{f_n} \sin^4 \phi_n \right. \\
 & \left. - \frac{2}{1-\nu_b} \left[\nu_b \beta_{\bullet LT_n} \nu_{f_n} + (1-2\nu_b) \beta'_{\bullet n} \nu_{f_n} \right] \sin^2 \phi_n \cos^2 \phi_n \right\} \\
 & + \frac{E_{f_n}(1-\nu_{f_n})}{1-\nu_{f_n}-2\nu_{f_n}} \left\{ \beta_{\bullet L_n} \nu_{f_n} \cos^4 \phi_n + \beta_{\bullet T_n} \nu_{f_n} \sin^4 \phi_n \right. \\
 & \left. + \frac{2}{1-\nu_{f_n}} \left[\nu_{f_n} \beta_{\bullet LT_n} \nu_{f_n} + (1-2\nu_{f_n}) \beta'_{\bullet n} \nu_{f_n} \right] \sin^2 \phi_n \cos^2 \phi_n \right\} \\
 A_2 = & \frac{E_b(1-\nu_b)}{1-\nu_b-2\nu_b} \left\{ \frac{\nu_b}{1-\nu_b} \left(\beta_{\bullet L_n} \nu_{f_n} + \beta_{\bullet T_n} \nu_{f_n} \right) \cos^2 \phi_n \cos^2 \psi_n \right. \\
 & - \frac{1}{1-\nu_b} \left[\nu_b \beta_{\bullet LT_n} \nu_{f_n} \left(\sin^2 \phi_n \cos^2 \psi_n + \cos^2 \phi_n \sin^2 \psi_n \right) \right. \\
 & \left. \left. + \nu_b \beta_{\bullet G_n} \nu_{f_n} \cos^2 \Omega_n - 2(1-2\nu_b) \beta'_{\bullet n} \nu_{f_n} \cos^2 \phi_n \cos^2 \psi_n \right] \right\} \\
 & + \frac{E_{f_n}(1-\nu_{f_n})}{1-\nu_{f_n}-2\nu_{f_n}} \left\{ \left(\beta_{\bullet L_n} \nu_{f_n} + \beta_{\bullet T_n} \nu_{f_n} \right) \cos^2 \phi_n \cos^2 \psi_n \right. \\
 & + \frac{1}{1-\nu_{f_n}} \left[\nu_{f_n} \beta_{\bullet LT_n} \nu_{f_n} \left(\sin^2 \phi_n \cos^2 \psi_n + \cos^2 \phi_n \sin^2 \psi_n \right) \right. \\
 & \left. \left. + \nu_{f_n} \beta_{\bullet G_n} \nu_{f_n} \cos^2 \Omega_n - 2(1-2\nu_{f_n}) \beta'_{\bullet n} \nu_{f_n} \cos^2 \phi_n \cos^2 \psi_n \right] \right\}
 \end{aligned}$$

TABLE 9. (continued)

$$\begin{aligned}
 A_3 = & \frac{E_b(1-\nu_b)}{1-\nu_b-2\nu_b^2} \left\{ \frac{\nu_b}{1-\nu_b} \left(\beta_{\bullet L_n} v_{f_n} + \beta_{\bullet T_n} v_{f_n} \right) \cos^2 \phi_n \cos^2 \Omega_n \right. \\
 & - \frac{1}{1-\nu_b} \left[\nu_b \beta_{\bullet LT_n} v_{f_n} \left(\sin^2 \phi_n \cos^2 \Omega_n + \cos^2 \phi_n \sin^2 \Omega_n \right) \right. \\
 & \left. \left. + \nu_b \beta_{\bullet G_n} v_{f_n} \cos^2 \psi_n - 2(1-2\nu_b) \beta'_{\bullet n} v_{f_n} \cos^2 \phi_n \cos^2 \Omega_n \right] \right\} \\
 & + \frac{E_{f_n}(1-\nu_{f_n})}{1-\nu_{f_n}-2\nu_{f_n}^2} \left\{ \left(\beta_{\bullet L_n} v_{f_n} + \beta_{\bullet T_n} v_{f_n} \right) \cos^2 \phi_n \cos^2 \Omega_n \right. \\
 & + \frac{1}{1-\nu_{f_n}} \left[\nu_{f_n} \beta_{\bullet LT_n} v_{f_n} \left(\sin^2 \phi_n \cos^2 \Omega_n + \cos^2 \phi_n \sin^2 \Omega_n \right) \right. \\
 & \left. \left. + \nu_{f_n} \beta_{\bullet G_n} v_{f_n} \cos^2 \psi_n - 2(1-2\nu_{f_n}) \beta'_{\bullet n} v_{f_n} \cos^2 \phi_n \cos^2 \Omega_n \right] \right\} \\
 A_4 = & \frac{E_b(1-\nu_b)}{1-\nu_b-2\nu_b^2} \left\{ 1 - \beta_{\bullet L_n} v_{f_n} \cos^4 \psi_n - \beta_{\bullet T_n} v_{f_n} \sin^4 \psi_n \right. \\
 & \left. - \frac{2}{1-\nu_b} \left[\nu_b \beta_{\bullet LT_n} v_{f_n} + (1-2\nu_b) \beta'_{\bullet n} v_{f_n} \right] \sin^2 \psi_n \cos^2 \psi_n \right\} \\
 & + \frac{E_{f_n}(1-\nu_{f_n})}{1-\nu_{f_n}-2\nu_{f_n}^2} \left\{ \beta_{\bullet L_n} v_{f_n} \cos^4 \psi_n + \beta_{\bullet T_n} v_{f_n} \sin^4 \psi_n \right. \\
 & \left. + \frac{2}{1-\nu_{f_n}} \left[\nu_{f_n} \beta_{\bullet LT_n} v_{f_n} + (1-2\nu_{f_n}) \beta'_{\bullet n} v_{f_n} \right] \sin^2 \psi_n \cos^2 \psi_n \right\}
 \end{aligned}$$

TABLE 9. (continued)

$$\begin{aligned}
 A_5 = & \frac{E_b(1-\nu_b)}{1-\nu_b-2\nu_b} \left\{ \frac{\nu_b}{1-\nu_b} - \left(\beta_{-L_n} v_{f_n} + \beta_{-T_n} v_{f_n} \right) \cos^2 \psi_n \cos^2 \Omega_n \right. \\
 & - \frac{1}{1-\nu_b} \left[\nu_b \beta_{-LT_n} v_{f_n} \left(\sin^2 \psi_n \cos^2 \Omega_n + \cos^2 \psi_n \sin^2 \Omega_n \right) \right. \\
 & \left. \left. + \nu_b \beta_{-G_n} v_{f_n} \cos^2 \phi_n - 2(1-2\nu_b) \beta'_{-n} v_{f_n} \cos^2 \psi_n \cos^2 \Omega_n \right] \right\} \\
 & + \frac{E_f(1-\nu_f)}{1-\nu_f-2\nu_f} \left\{ \left(\beta_{L_n} v_{f_n} + \beta_{T_n} v_{f_n} \right) \cos^2 \psi_n \cos^2 \Omega_n \right. \\
 & + \frac{1}{1-\nu_f} \left[\nu_f \beta_{LT_n} v_{f_n} \left(\sin^2 \psi_n \cos^2 \Omega_n + \cos^2 \psi_n \sin^2 \Omega_n \right) \right. \\
 & \left. \left. + \nu_f \beta_{G_n} v_{f_n} \cos^2 \phi_n - 2(1-2\nu_f) \beta'_{n} v_{f_n} \cos^2 \psi_n \cos^2 \Omega_n \right] \right\} \\
 A_6 = & \frac{E_b(1-\nu_b)}{1-\nu_b-2\nu_b} \left\{ 1 - \beta_{-L_n} v_{f_n} \cos^4 \Omega_n - \beta_{-T_n} v_{f_n} \sin^4 \Omega_n \right. \\
 & \left. - \frac{2}{1-\nu_b} \left[\nu_b \beta_{-LT_n} v_{f_n} + (1-2\nu_b) \beta'_{-n} v_{f_n} \right] \sin^2 \Omega_n \cos^2 \Omega_n \right\} \\
 & + \frac{E_f(1-\nu_f)}{1-\nu_f-2\nu_f} \left\{ \beta_{L_n} v_{f_n} \cos^4 \Omega_n + \beta_{T_n} v_{f_n} \sin^4 \Omega_n \right. \\
 & \left. + \frac{2}{1-\nu_f} \left[\nu_f \beta_{LT_n} v_{f_n} + (1-2\nu_f) \beta'_{n} v_{f_n} \right] \sin^2 \Omega_n \cos^2 \Omega_n \right\}
 \end{aligned}$$

TABLE 9 (continued)

$$\begin{aligned}
 A_7 = G_b \left\{ \right. & 1 - \beta'_{-\bullet n} v_{f_n} \sin^2 \Omega_n - \beta''_{-\bullet n} v_{f_n} \cos^2 \Omega_n \\
 & - 4 \left[\frac{1 - \nu_b}{1 - 2\nu_b} \left(\frac{\beta_{-\bullet L_n} v_{f_n} + \beta_{-\bullet T_n} v_{f_n}}{2} \right) \right. \\
 & \quad \left. \left. - \frac{\nu_b}{1 - 2\nu_b} \left(\beta_{-\bullet LT_n} v_{f_n} \right) - \beta'_{-\bullet n} v_{f_n} \right] \cos^2 \phi_n \cos^2 \psi_n \right\} \\
 + G_{f_n} \left\{ \right. & \beta'_{\bullet n} v_{f_n} \sin^2 \Omega_n + \beta''_{\bullet n} v_{f_n} \cos^2 \Omega_n \\
 & + 4 \left[\frac{1 - \nu_{f_n}}{1 - 2\nu_{f_n}} \left(\frac{\beta_{\bullet L_n} v_{f_n} + \beta_{\bullet T_n} v_{f_n}}{2} \right) \right. \\
 & \quad \left. \left. - \frac{\nu_{f_n}}{1 - 2\nu_{f_n}} \left(\beta_{\bullet LT_n} v_{f_n} \right) - \beta'_{\bullet n} v_{f_n} \right] \cos^2 \phi_n \cos^2 \psi_n \right\} \\
 A_8 = G_b \left\{ \right. & 1 - \beta'_{-\bullet n} v_{f_n} \sin^2 \phi_n - \beta''_{-\bullet n} v_{f_n} \cos^2 \phi_n \\
 & - 4 \left[\frac{1 - \nu_b}{1 - 2\nu_b} \left(\frac{\beta_{-\bullet L_n} v_{f_n} + \beta_{-\bullet T_n} v_{f_n}}{2} \right) - \frac{\nu_b}{1 - 2\nu_b} \left(\beta_{-\bullet LT_n} v_{f_n} \right) \right. \\
 & \quad \left. \left. - \beta'_{-\bullet n} v_{f_n} \right] \cos^2 \psi_n \cos^2 \Omega_n \right\} \\
 + G_{f_n} \left\{ \right. & \beta'_{\bullet n} v_{f_n} \sin^2 \phi_n + \beta''_{\bullet n} v_{f_n} \cos^2 \phi_n \\
 & + 4 \left[\frac{1 - \nu_{f_n}}{1 - 2\nu_{f_n}} \left(\frac{\beta_{\bullet L_n} v_{f_n} + \beta_{\bullet T_n} v_{f_n}}{2} \right) - \frac{\nu_{f_n}}{1 - 2\nu_{f_n}} \left(\beta_{\bullet LT_n} v_{f_n} \right) \right. \\
 & \quad \left. \left. - \beta'_{\bullet n} v_{f_n} \right] \cos^2 \psi_n \cos^2 \Omega_n \right\}
 \end{aligned}$$

TABLE 9. Generalized Equations for "Stiffnesses" of Three-Dimensionally Reinforced Filamentary Composite in Terms of the Transverse Effectiveness Coefficients (β).

$$\begin{aligned}
 A_1 = & \frac{E_b(1-\nu_b)}{1-\nu_b-2\nu_b^2} \left\{ 1-\beta_{\bullet L_n} \nu_{f_n} \cos^4 \phi_n - \beta_{\bullet T_n} \nu_{f_n} \sin^4 \phi_n \right. \\
 & \left. - \frac{2}{1-\nu_b} \left[\nu_b \beta_{\bullet LT_n} \nu_{f_n} + (1-2\nu_b) \beta'_{\bullet n} \nu_{f_n} \right] \sin^2 \phi_n \cos^2 \phi_n \right\} \\
 & + \frac{E_{f_n}(1-\nu_{f_n})}{1-\nu_{f_n}-2\nu_{f_n}^2} \left\{ \beta_{\bullet L_n} \nu_{f_n} \cos^4 \phi_n + \beta_{\bullet T_n} \nu_{f_n} \sin^4 \phi_n \right. \\
 & \left. + \frac{2}{1-\nu_{f_n}} \left[\nu_{f_n} \beta_{\bullet LT_n} \nu_{f_n} + (1-2\nu_{f_n}) \beta'_{\bullet n} \nu_{f_n} \right] \sin^2 \phi_n \cos^2 \phi_n \right\} \\
 A_2 = & \frac{E_b(1-\nu_b)}{1-\nu_b-2\nu_b^2} \left\{ \frac{\nu_b}{1-\nu_b} \left(\beta_{\bullet L_n} \nu_{f_n} + \beta_{\bullet T_n} \nu_{f_n} \right) \cos^2 \phi_n \cos^2 \psi_n \right. \\
 & - \frac{1}{1-\nu_b} \left[\nu_b \beta_{\bullet LT_n} \nu_{f_n} \left(\sin^2 \phi_n \cos^2 \psi_n + \cos^2 \phi_n \sin^2 \psi_n \right) \right. \\
 & \left. \left. + \nu_b \beta_{\bullet G_n} \nu_{f_n} \cos^2 \Omega_n - 2(1-2\nu_b) \beta'_{\bullet n} \nu_{f_n} \cos^2 \phi_n \cos^2 \psi_n \right] \right\} \\
 & + \frac{E_{f_n}(1-\nu_{f_n})}{1-\nu_{f_n}-2\nu_{f_n}^2} \left\{ \left(\beta_{\bullet L_n} \nu_{f_n} + \beta_{\bullet T_n} \nu_{f_n} \right) \cos^2 \phi_n \cos^2 \psi_n \right. \\
 & + \frac{1}{1-\nu_{f_n}} \left[\nu_{f_n} \beta_{\bullet LT_n} \nu_{f_n} \left(\sin^2 \phi_n \cos^2 \psi_n + \cos^2 \phi_n \sin^2 \psi_n \right) \right. \\
 & \left. \left. + \nu_{f_n} \beta_{\bullet G_n} \nu_{f_n} \cos^2 \Omega_n - 2(1-2\nu_{f_n}) \beta'_{\bullet n} \nu_{f_n} \cos^2 \phi_n \cos^2 \psi_n \right] \right\}
 \end{aligned}$$

TABLE 10. General Expression for the Potential Energy of a Three-Dimensionally Reinforced Filamentary Composite Subjected to Strains $\epsilon_1, \epsilon_2, \epsilon_3, \gamma_{12}, \gamma_{23}, \gamma_{13}$, and a Temperature Rise ΔT .

$$\begin{aligned}
 V = & \frac{1}{2} \int_0^{b_n} \left\{ \frac{E_b}{1-\nu_b-2\nu_b^2} \right\} \left\{ [1-\nu_b] \left[\epsilon_1^2 + \epsilon_2^2 + \epsilon_3^2 - 2\alpha_b \Delta T (\epsilon_1 + \epsilon_2 + \epsilon_3) \right] \right. \\
 & + [2\nu_b] \left[(\epsilon_1 \epsilon_2 + \epsilon_2 \epsilon_3 + \epsilon_1 \epsilon_3) - 2\alpha_b \Delta T (\epsilon_1 + \epsilon_2 + \epsilon_3) \right] \\
 & \left. + \left[\frac{1-2\nu_b}{2} \right] \left[\gamma_{12}^2 + \gamma_{23}^2 + \gamma_{13}^2 \right] \right\} d(n) \\
 & + \frac{1}{2} \int_0^{b_n} \left\{ \frac{E_{f_n} (1-\nu_{f_n}) \beta_{L_n} \nu_{f_n}^V}{1-\nu_{f_n}-2\nu_{f_n}^2} - \frac{E_b (1-\nu_b) \beta_{L_n} \nu_{f_n}^V}{1-\nu_b-2\nu_b^2} \right\} \left\{ \epsilon_n^2 \right\} d(n) \\
 & - \frac{1}{2} \int_0^{b_n} \left\{ \frac{\alpha_{f_n} E_{f_n} (1-\nu_{f_n}) \beta_{L_n} \nu_{f_n}^V}{1-\nu_{f_n}-2\nu_{f_n}^2} - \frac{\alpha_b E_b (1-\nu_b) \beta_{L_n} \nu_{f_n}^V}{1-\nu_b-2\nu_b^2} \right\} \left\{ 2\Delta T \epsilon_n \right\} d(n) \\
 & + \frac{1}{2} \int_0^{b_n} \left\{ \frac{\nu_{f_n} E_{f_n} \beta_{L_n} \nu_{f_n}^V}{1-\nu_{f_n}-2\nu_{f_n}^2} - \frac{\nu_b E_b \beta_{L_n} \nu_{f_n}^V}{1-\nu_b-2\nu_b^2} \right\} \left\{ \begin{array}{l} [\epsilon_1 \epsilon_2 + \epsilon_1 \epsilon_3]_{n=1} \\ [\epsilon_1 \epsilon_2 + \epsilon_2 \epsilon_3]_{n=2} \\ [\epsilon_2 \epsilon_3 + \epsilon_1 \epsilon_3]_{n=3} \end{array} \right\} d(n) \\
 & - \frac{1}{2} \int_0^{b_n} \left\{ \frac{\alpha_{f_n} \nu_{f_n} E_{f_n} \beta_{L_n} \nu_{f_n}^V}{1-\nu_{f_n}-2\nu_{f_n}^2} - \frac{\alpha_b \nu_b E_b \beta_{L_n} \nu_{f_n}^V}{1-\nu_b-2\nu_b^2} \right\} \left\{ \begin{array}{l} \left[\epsilon_1 + \frac{\epsilon_2 + \epsilon_3}{2} \right] [2\Delta T]_{n=1} \\ \left[\epsilon_2 + \frac{\epsilon_1 + \epsilon_3}{2} \right] [2\Delta T]_{n=2} \\ \left[\epsilon_3 + \frac{\epsilon_1 + \epsilon_2}{2} \right] [2\Delta T]_{n=3} \end{array} \right\} d(n)
 \end{aligned}$$

TABLE 10. (continued)

$$+ \frac{1}{2} \int_0^{b_n} \left\{ \frac{E_{f_n} (1-\nu_{f_n}) \beta_{\bullet T_n} V_{f_n}}{1-\nu_{f_n} - 2\nu_{f_n}^2} - \frac{E_b (1-\nu_b) \beta_{\bullet T_n} V_{f_n}}{1-\nu_b - 2\nu_b^2} \right\} \left\{ \begin{array}{l} [\epsilon_2^2 + \epsilon_3^2]_{n=1} \\ [\epsilon_1^2 + \epsilon_3^2]_{n=2} \\ [\epsilon_1^2 + \epsilon_2^2]_{n=3} \end{array} \right\} d(n)$$

$$- \frac{1}{2} \int_0^{b_n} \left\{ \frac{\alpha_{f_n} E_{f_n} (1-\nu_{f_n}) \beta_{\bullet T_n} V_{f_n}}{1-\nu_{f_n} - 2\nu_{f_n}^2} - \frac{\alpha_b E_b (1-\nu_b) \beta_{\bullet T_n} V_{f_n}}{1-\nu_b - 2\nu_b^2} \right\} \left\{ \begin{array}{l} [\epsilon_2 + \epsilon_3] [2\Delta T]_{n=1} \\ [\epsilon_1 + \epsilon_3] [2\Delta T]_{n=2} \\ [\epsilon_1 + \epsilon_2] [2\Delta T]_{n=3} \end{array} \right\} d(n)$$

$$+ \frac{1}{2} \int_0^{b_n} \left\{ \frac{\nu_{f_n} E_{f_n} \beta_{\bullet G_n} V_{f_n}}{1-\nu_{f_n} - 2\nu_{f_n}^2} - \frac{\nu_b E_b \beta_{\bullet G_n} V_{f_n}}{1-\nu_b - 2\nu_b^2} \right\} \left\{ \begin{array}{l} [\epsilon_2 \epsilon_3]_{n=1} \\ [\epsilon_1 \epsilon_3]_{n=2} \\ [\epsilon_1 \epsilon_2]_{n=3} \end{array} \right\} d(n)$$

$$- \frac{1}{2} \int_0^{b_n} \left\{ \frac{\alpha_{f_n} \nu_{f_n} E_{f_n} \beta_{\bullet G_n} V_{f_n}}{1-\nu_{f_n} - 2\nu_{f_n}^2} - \frac{\alpha_b \nu_b E_b \beta_{\bullet G_n} V_{f_n}}{1-\nu_b - 2\nu_b^2} \right\} \left\{ \begin{array}{l} \left[\frac{\epsilon_2 + \epsilon_3}{2} \right] [2\Delta T]_{n=1} \\ \left[\frac{\epsilon_1 + \epsilon_3}{2} \right] [2\Delta T]_{n=2} \\ \left[\frac{\epsilon_1 + \epsilon_2}{2} \right] [2\Delta T]_{n=3} \end{array} \right\} d(n)$$

$$+ \frac{1}{2} \int_0^{b_n} \left\{ G_{f_n} \beta'_{\bullet n} V_{f_n} - G_b \beta'_{\bullet n} V_{f_n} \right\} \left\{ \gamma_{f_n LL}^2 + \gamma_{f_n LT}^2 \right\} d(n)$$

$$+ \frac{1}{2} \int_0^{b_n} \left\{ G_{f_n} \beta''_{\bullet n} V_{f_n} - G_b \beta''_{\bullet n} V_{f_n} \right\} \left\{ \gamma_{f_n TT}^2 \right\} d(n)$$

TABLE 10. (continued)

$$\begin{aligned}
 & + \frac{1}{2} \int_0^{b_s} \left\{ \frac{E_{f_s} (1-\nu_{f_s}) \beta_{\bullet L_s} V_{f_s}}{1-\nu_{f_s} - 2\nu_{f_s}^2} - \frac{E_b (1-\nu_b) \beta_{\bullet L_s} V_{f_s}}{1-\nu_b - 2\nu_b^2} \right\} \left\{ \epsilon_{sL}^2 \right\} d(s) \\
 & - \frac{1}{2} \int_0^{b_s} \left\{ \frac{\alpha_{f_s} E_{f_s} (1-\nu_{f_s}) \beta_{\bullet L_s} V_{f_s}}{1-\nu_{f_s} - 2\nu_{f_s}^2} - \frac{\alpha_b E_b (1-\nu_b) \beta_{\bullet L_s} V_{f_s}}{1-\nu_b - 2\nu_b^2} \right\} \left\{ \epsilon_{sL} (2\Delta T) \right\} d(s) \\
 & + \frac{1}{2} \int_0^{b_s} \left\{ \frac{\nu_{f_s} E_{f_s} \beta_{\bullet LT_s} V_{f_s}}{1-\nu_{f_s} - 2\nu_{f_s}^2} - \frac{\nu_b E_b \beta_{\bullet LT_s} V_{f_s}}{1-\nu_b - 2\nu_b^2} \right\} \left\{ \epsilon_{sL} \epsilon_{sT_1} + \epsilon_{sL} \epsilon_{sT_2} \right\} d(s) \\
 & - \frac{1}{2} \int_0^{b_s} \left\{ \frac{\alpha_{f_s} \nu_{f_s} E_{f_s} \beta_{\bullet LT_s} V_{f_s}}{1-\nu_{f_s} - 2\nu_{f_s}^2} - \frac{\alpha_b \nu_b E_b \beta_{\bullet LT_s} V_{f_s}}{1-\nu_b - 2\nu_b^2} \right\} \left\{ \epsilon_{sL} + \frac{\epsilon_{sT_1} + \epsilon_{sT_2}}{2} (2\Delta T) \right\} d(s) \\
 & + \frac{1}{2} \int_0^{b_s} \left\{ \frac{E_{f_s} (1-\nu_{f_s}) \beta_{\bullet T_s} V_{f_s}}{1-\nu_{f_s} - 2\nu_{f_s}^2} - \frac{E_b (1-\nu_b) \beta_{\bullet T_s} V_{f_s}}{1-\nu_b - 2\nu_b^2} \right\} \left\{ \epsilon_{sT_1}^2 + \epsilon_{sT_2}^2 \right\} d(s) \\
 & - \frac{1}{2} \int_0^{b_s} \left\{ \frac{\alpha_{f_s} E_{f_s} (1-\nu_{f_s}) \beta_{\bullet T_s} V_{f_s}}{1-\nu_{f_s} - 2\nu_{f_s}^2} - \frac{\alpha_b E_b (1-\nu_b) \beta_{\bullet T_s} V_{f_s}}{1-\nu_b - 2\nu_b^2} \right\} \left\{ (\epsilon_{sT_1} + \epsilon_{sT_2}) (2\Delta T) \right\} d(s) \\
 & + \frac{1}{2} \int_0^{b_s} \left\{ \frac{\nu_{f_s} E_{f_s} \beta_{\bullet G_s} V_{f_s}}{1-\nu_{f_s} - 2\nu_{f_s}^2} - \frac{\nu_b E_b \beta_{\bullet G_s} V_{f_s}}{1-\nu_b - 2\nu_b^2} \right\} \left\{ \epsilon_{sT_1} \epsilon_{sT_2} \right\} d(s)
 \end{aligned}$$

TABLE 10. (concluded)

$$\begin{aligned}
 & -\frac{1}{2} \int_0^{b_s} \left\{ \frac{\alpha_{f_s} \nu_{f_s} E_{f_s} \beta_{f_s} \bullet G_{f_s} V_{f_s}}{1 - \nu_{f_s} - 2\nu_{f_s}^2} - \frac{\alpha_{b_s} \nu_{b_s} E_{b_s} \beta_{b_s} \bullet G_{b_s} V_{f_s}}{1 - \nu_{b_s} - 2\nu_{b_s}^2} \right\} \left(\frac{\epsilon_{sT_1} + \epsilon_{sT_2}}{2} \right) (2\Delta T) \Bigg\} d(s) \\
 & + \frac{1}{2} \int_0^{b_s} \left\{ G_{f_s} \beta'_{f_s} \bullet V_{f_s} - G_{b_s} \beta'_{b_s} \bullet V_{f_s} \right\} \left\{ \gamma_{sLL}^2 + \gamma_{sLT}^2 \right\} d(s) \\
 & + \frac{1}{2} \int_0^{b_s} \left\{ G_{f_s} \beta''_{f_s} \bullet V_{f_s} - G_{b_s} \beta''_{b_s} \bullet V_{f_s} \right\} \left\{ \gamma_{sTT}^2 \right\} d(s)
 \end{aligned}$$

TABLE 11. Values of Thermal Expansion Coefficients Calculated by 2-D Laminate Analysis (LILAC II) and 3-D Analysis for Tungsten/Epoxy Composite with Uni-directional Reinforcement. $\nu_f=0.2$

<u>Constituent Properties Assumed</u>						
	$E_b = 500 \text{ ksi } (3.45 \text{ GN/m}^2)$			$E_f = 50,000 \text{ ksi } (345 \text{ GN/m}^2)$		
	$\nu_b = 0.35$			$\nu_f = 0.30, 0.35, \text{ and } 0.40$		
	$\alpha_b = 25 \times 10^{-6} / ^\circ\text{F } (45 \times 10^{-6} / ^\circ\text{K})$			$\alpha_f = 2.5 \times 10^{-6} / ^\circ\text{F } (4.5 \times 10^{-6} / ^\circ\text{K})$		
<u>Calculated Thermal Expansion Coefficients</u>						
	$\alpha_1 \times 10^6 / ^\circ\text{F}$ (/ $^\circ\text{K}$)			$\alpha_2 \times 10^6 / ^\circ\text{F}$ (/ $^\circ\text{K}$)		
	$\nu_f = 0.30$		0.35	0.40	$\nu_f = 0.30$	
					0.35 0.40	
LILAC II	3.3816416 (6.086955)	3.3653846 (6.0576923)	3.3490594 (6.028307)	25.14833 (45.26699)	25.138475 (45.249255)	25.128605 (45.23149)
3-D	3.4119391 (6.1414904)	3.3653846 (6.0576923)	3.2499306 (5.849875)	25.14031 (45.25256)	25.138476 (45.249257)	25.154961 (45.27893)
<u>Check of Average Expansions</u>						
	$\frac{\alpha_1 + \alpha_2 + \alpha_3}{3}$					
	$\nu_f = 0.30$		0.35	0.40		
LILAC II	17.892767 (32.206981)		17.880778 (32.185400)		17.868756 (32.163761)	
3-D	17.897520 (32.215537)		17.880778 (32.185400)		17.853284 (32.135911)	
	$\bar{\alpha}^*$					
	17.892768 (32.206982)		17.880778 (32.185400)		17.868756 (32.163761)	

TABLE 12. Values of Thermal Expansion Coefficients α_1 and α_2 [$^{\circ}\text{F}$ or ($^{\circ}\text{K}$) $\times 10^{-6}$] for Two- and Three-Dimensional Reinforcement Configurations of Boron/Epoxy of Properties Given in Table 1.

		$\pm \theta$, deg.											
Configuration	v_f	0		22 1/2		27 1/2		32 1/2		37 1/2		45	
		α_1	α_2	α_1	α_2	α_1	α_2	α_1	α_2	α_1	α_2	α_1	α_2
2-D	0.2	5.356 (9.64)	25.17 (45.3)	2.832 (5.10)	23.19 (41.7)	2.154 (3.88)	21.08 (37.9)	1.460 (2.63)	18.63 (33.5)	2.797 (5.03)	13.78 (24.8)	7.029 (12.7)	7.029 (12.7)
3-D	.2	5.455 (9.82)	25.11 (45.2)	2.609 (4.70)	25.34 (45.6)	1.112 (2.00)	25.27 (45.5)	-0.748 (-1.35)	25.03 (45.1)	-2.258 (-4.06)	23.98 (43.2)	-2.137 (-3.85)	20.12 (36.2)
2-D	.4	4.904 (8.83)	19.30 (34.7)	2.963 (5.33)	17.99 (32.4)	2.215 (3.99)	16.78 (30.2)	1.782 (3.21)	14.73 (26.5)	2.259 (4.07)	11.62 (20.9)	5.895 (10.6)	5.895 (10.6)
3-D	.4	5.004 (9.01)	19.21 (34.6)	2.866 (5.16)	19.30 (34.7)	1.730 (3.11)	19.21 (34.6)	0.395 (0.711)	18.88 (34.0)	-0.898 (-1.62)	18.07 (32.5)	-1.003 (-1.81)	15.00 (27.0)
2-D	.6	4.742 (8.54)	13.93 (25.1)	3.514 (6.33)	13.16 (23.7)	3.039 (5.47)	12.42 (22.4)	2.765 (4.98)	11.14 (2.01)	3.081 (5.55)	9.166 (16.5)	5.459 (9.83)	5.459 (9.83)
3-D	.6	4.838 (8.71)	13.85 (24.9)	3.514 (6.33)	13.84 (24.9)	2.826 (5.09)	13.74 (24.7)	2.040 (3.67)	13.47 (24.2)	1.329 (2.39)	12.88 (23.2)	1.448 (2.61)	10.88 (19.6)

TABLE 13. Values of Elastic Constants, E_1 [ksi or (GN/m^2)] for Two- and Three- Dimensional Reinforcement Configurations of Boron/Epoxy of Properties Given in Table 1.

Configuration	v_f	$\pm \theta$, deg.											
		0		22 1/2		27 1/2		32 1/2		37 1/2		45	
		E_1	E_2	E_1	E_2	E_1	E_2	E_1	E_2	E_1	E_2	E_1	E_2
2-D	0.2	11,402 (78.7)	818.5 (5.65)	6164 (42.5)	753.4 (5.20)	4192 (28.9)	737.0 (5.09)	2686 (18.5)	738.2 (5.09)	1723 (11.9)	779.9 (5.38)	1016 (7.01)	1016 (7.01)
3-D	.2	11,402 (78.7)	818.5 (5.65)	7222 (49.8)	906.8 (6.26)	5524 (38.1)	997.6 (6.88)	3932 (27.1)	1110 (7.66)	2637 (18.2)	1223 (8.44)	1395 (9.63)	1387 (9.57)
2-D	.4	22,304 (153.9)	1309 (9.03)	11,395 (78.6)	1193 (8.23)	7419 (51.2)	1160 (8.00)	4545 (31.4)	1154 (7.96)	2802 (19.3)	1213 (8.37)	1592 (11.0)	1592 (11.0)
3-D	.4	22,304 (153.9)	1309 (9.03)	13,783 (95.1)	1459 (10.1)	10,276 (70.9)	1613 (11.1)	7077 (48.8)	1798 (12.4)	4574 (31.6)	1976 (13.6)	2308 (15.9)	2250 (15.5)
2-D	.6	33,203 (229.1)	2286 (15.8)	17,757 (122.5)	2066 (14.3)	11,855 (81.8)	1999 (13.8)	7413 (51.1)	1976 (13.6)	4634 (32.0)	2059 (14.2)	2667 (18.4)	2667 (18.4)
3-D	.6	33,203 (229.1)	2286 (15.8)	20,855 (143.9)	2453 (16.9)	15,725 (108.5)	2652 (18.3)	10,997 (75.9)	2906 (20.1)	7260 (50.1)	3176 (21.9)	3816 (26.3)	3653 (25.2)

TABLE 14. Values of Tensile Strength to Density Ratio and Axial Stiffness to Density Ratio of Zero Expansion Composites, and of Titanium Alloy and Aluminum Alloy for Comparison

Material	v_f	ρ , pci ($\frac{\text{Mg}}{\text{m}^3}$)	$\left(\frac{\sigma_{tu}}{\rho}\right)$, in. $\left(\frac{\text{Nm}}{\text{g}}\right)$	$\left(\frac{E_1}{\rho}\right)$, in $\left(\frac{\text{kNm}}{\text{g}}\right)$
2-D "Tungsten"/ "Polyester" $\alpha_1 \approx 0$	0.2	0.1762 (4.88)	480,000 (119.7)	25,870,000 (6.45)
			170,300 (42.46)	9,874,970 (2.46)
	0.3	0.2413 (6.68)	386,700 (96.40)	26,980,000 (6.73)
			120,200 (30.0)	8,790,000 (2.19)
	0.4	0.3064 (8.48)	255,200 (63.6)	23,610,000 (5.89)
		99,540 (24.8)	9,240,000 (2.30)	
	0.5	0.3715 (10.28)	110,900 (27.6)	13,520,000 (3.37)
3-D Boron/Epoxy $\alpha_1 \approx 0$	0.4	0.0664 (1.84)	2,364,000 (589.3)	91,520,000 (22.8)
Ti-5Al-2.5 Sn $\alpha = 5.2 \times 10^{-6}/^{\circ}\text{F}$ ($9.4 \times 10^{-6}/^{\circ}\text{K}$)	-	0.161 (4.46)	869,600 (216.6)	102,500,000 (25.6)
7075-T6 $\alpha = 11.7 \times 10^{-6}/^{\circ}\text{F}$ ($21.1 \times 10^{-6}/^{\circ}\text{K}$)	-	0.101 (2.80)	792,000 (197.4)	104,000,000 (25.9)

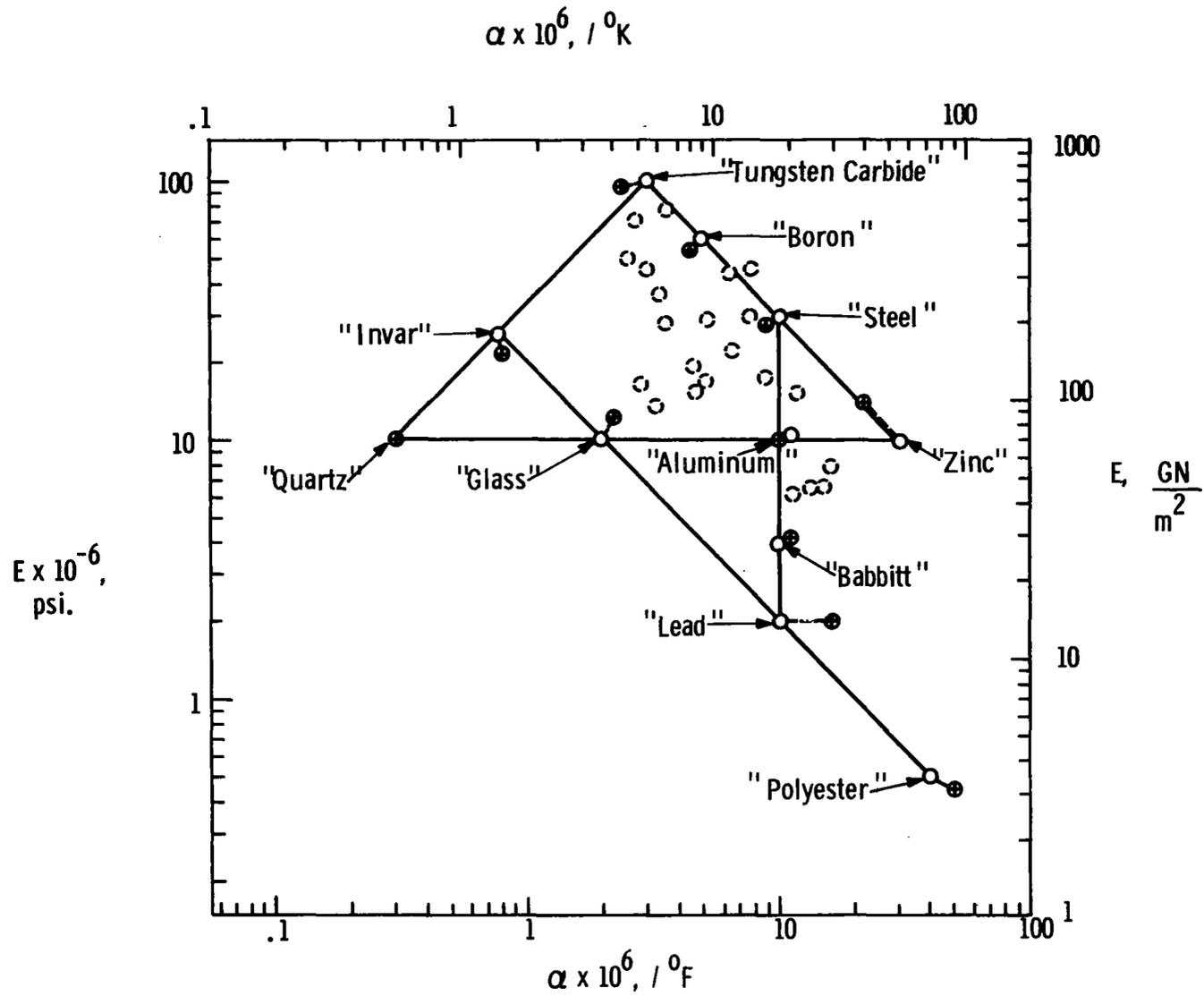
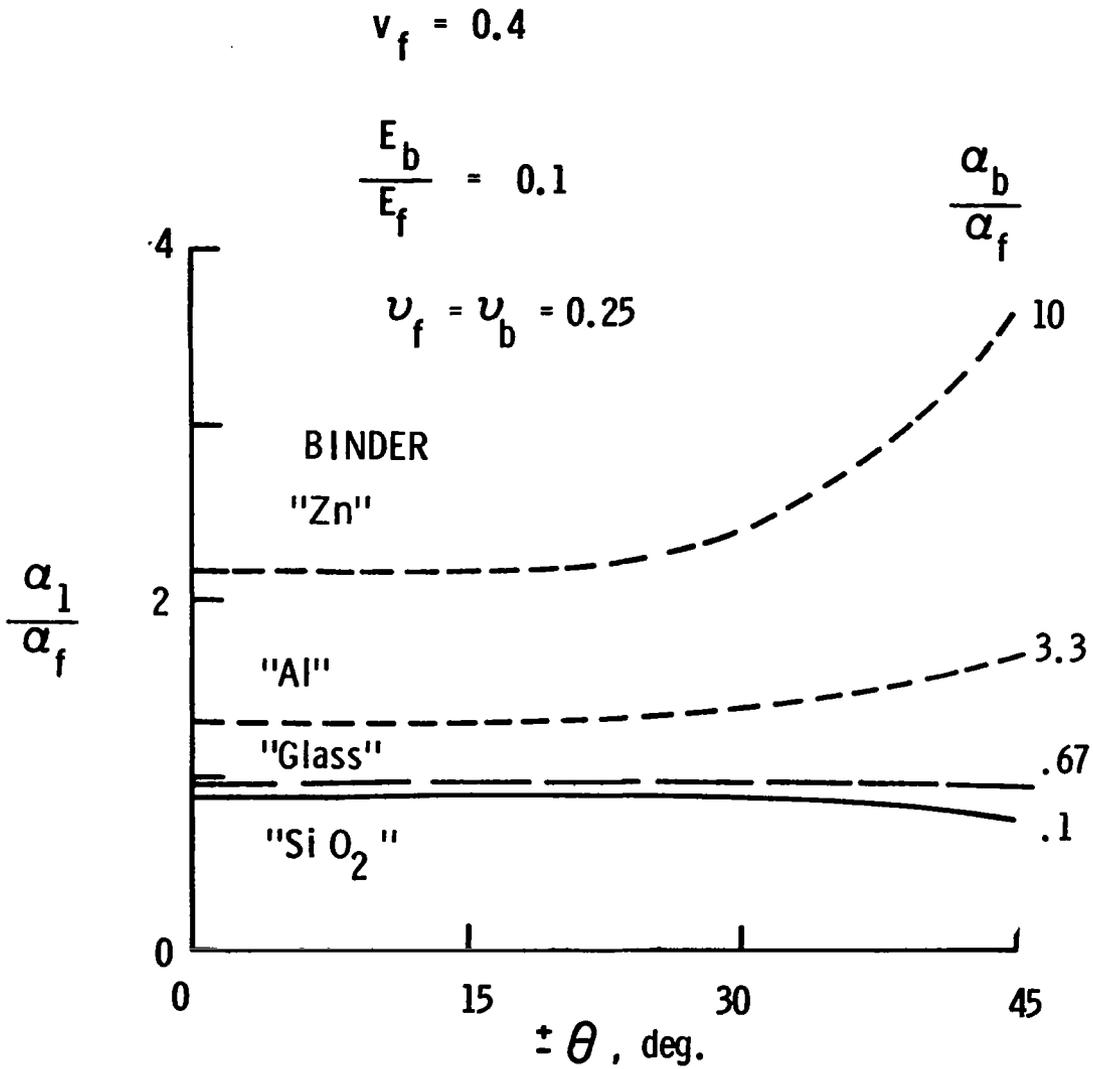


Figure 1. Young's modulus and coefficient of thermal expansion of various materials (from Table 1), showing selected, slightly idealized materials used for study.

"TUNGSTEN - CARBIDE" FILAMENTS



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Figure 2. Ratio of coefficient of thermal expansion in the 1-direction to coefficient of thermal expansion of the filaments for two-dimensional reinforcement at angles of $\pm \theta$ to the 1-direction. Case Ia: Binders of constant Young's modulus and varying coefficients of thermal expansion.

"POLYESTER BINDER"

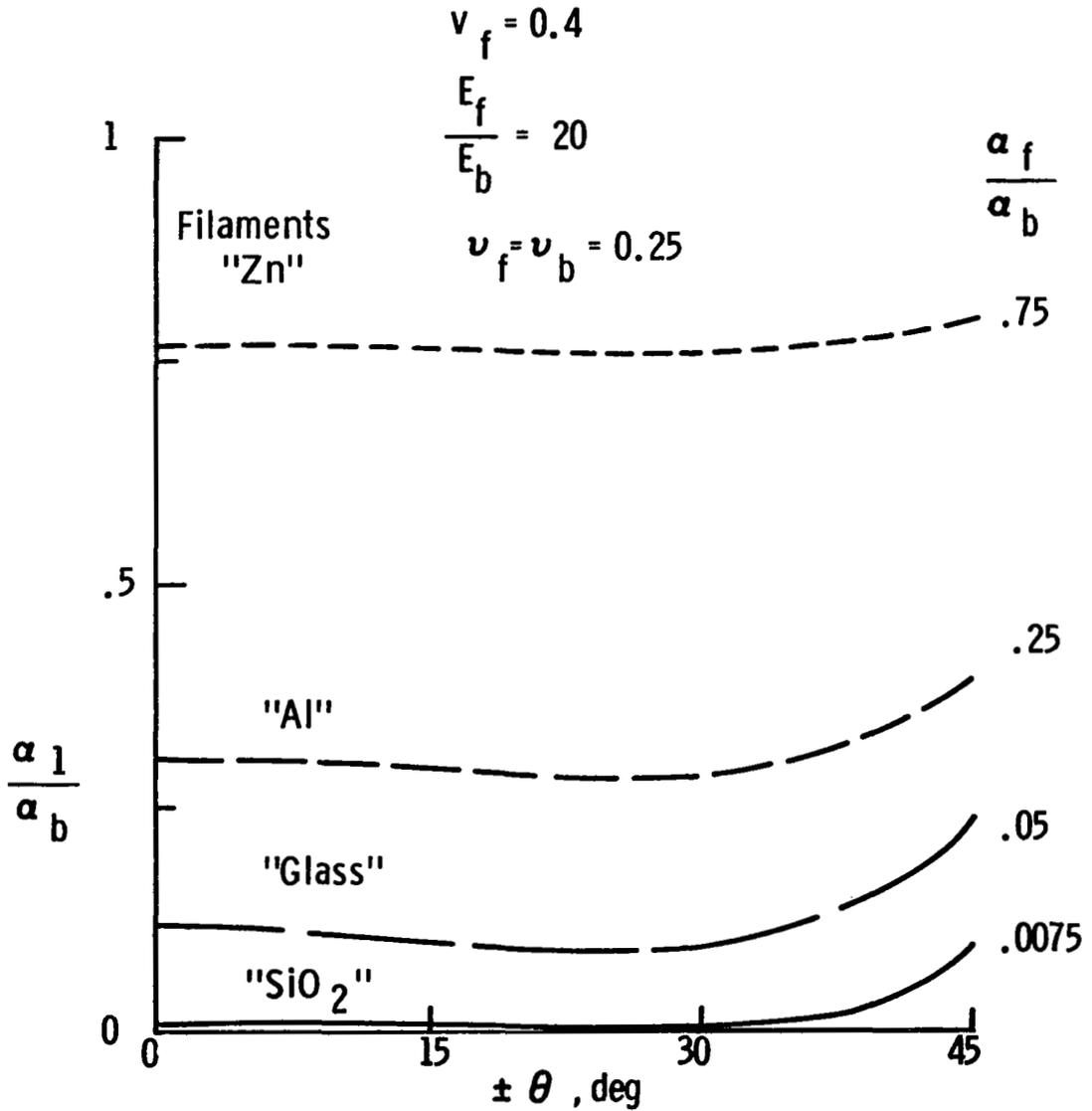


Figure 3. Ratio of coefficient of thermal expansion in the 1-direction to coefficient of thermal expansion of the binder for two-dimensional reinforcement at angles of $\pm \theta$ to the 1-direction. Case 1b. Filaments of constant Young's modulus and varying coefficients of thermal expansion.

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$$\nu_f = 0.4$$

$$\frac{\alpha_f}{\alpha_b} = 0.3$$

$$\nu_f = \nu_b = 0.25$$

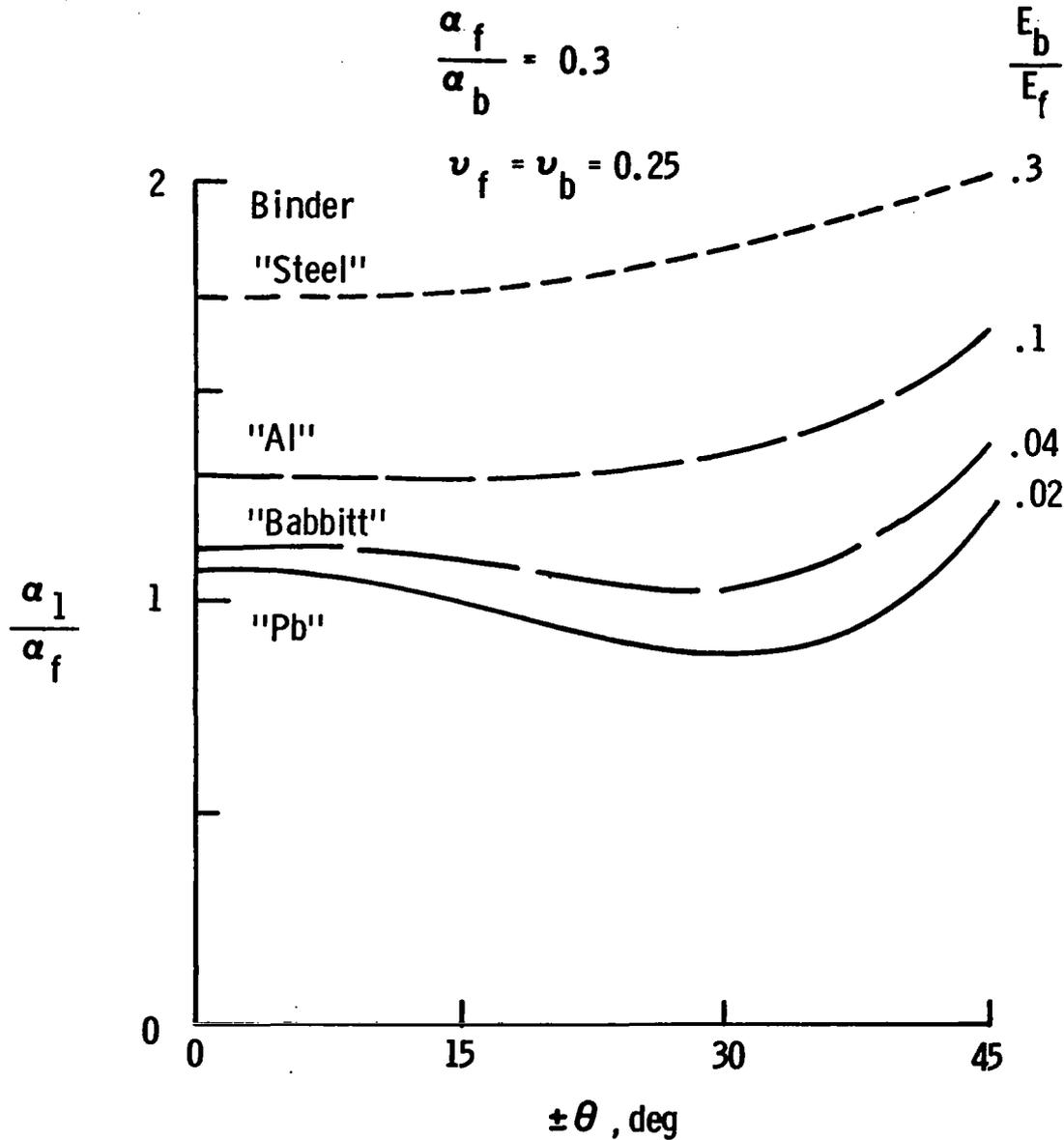


Figure 4. Ratio of coefficient of thermal expansion in the 1-direction to coefficient of thermal expansion of the filaments for two-dimensional reinforcement at angles of $\pm \theta$ to the 1-direction. Case II. Binders of constant coefficient of thermal expansion and varying Young's modulus.

"POLYESTER" BINDER

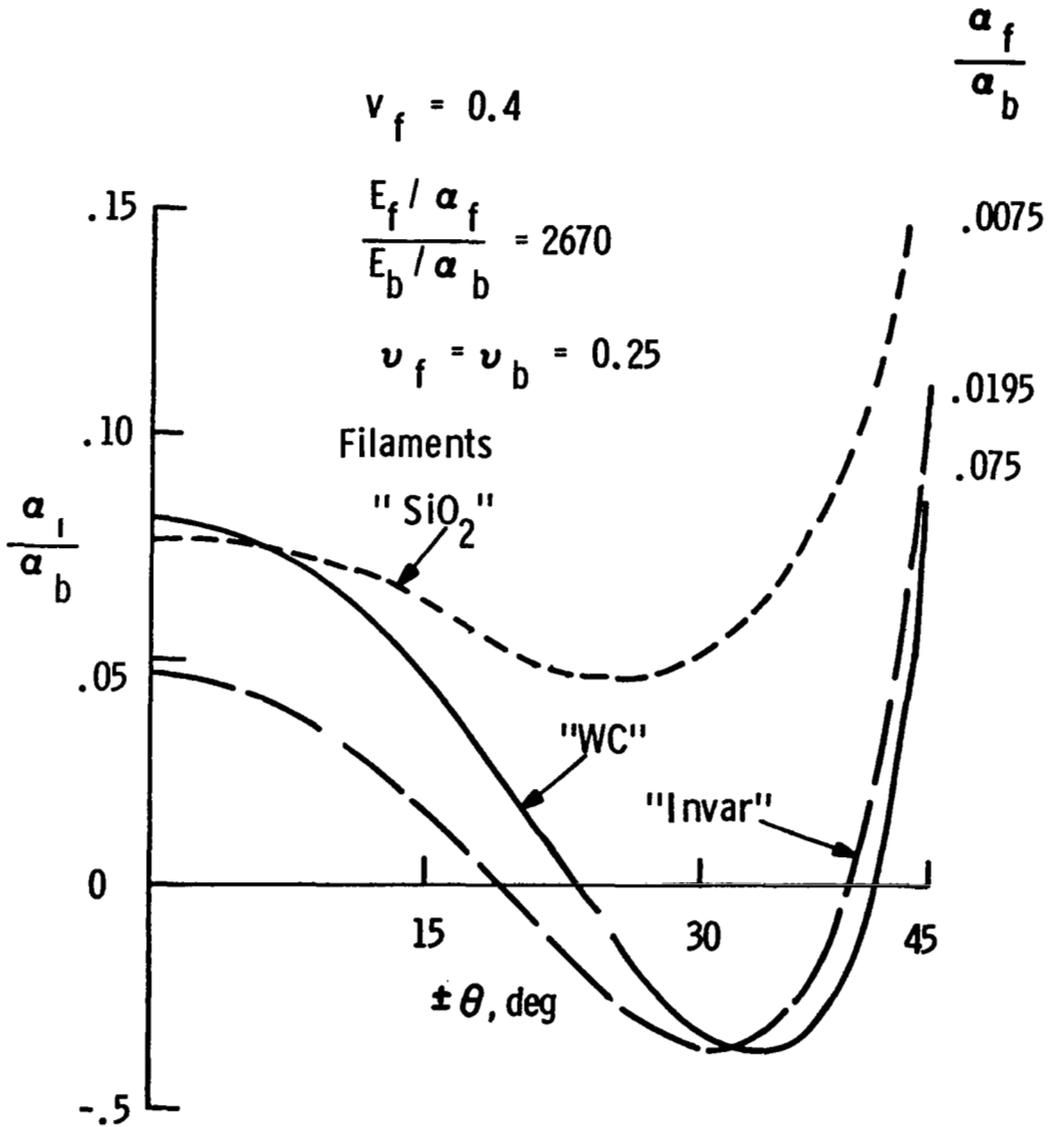


Figure 5. Ratio of coefficient of thermal expansion in the 1-direction to coefficient of thermal expansion of the binder for two-dimensional reinforcement at angles $\pm \theta$ to the 1-direction. Case V. Filaments of constant ratio of Young's modulus to coefficient of thermal expansion.

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$$\nu_f = 0.4$$

$$\frac{\alpha_b E_b}{\alpha_f E_f} = .067$$

$$\nu_f = \nu_b = 0.25$$

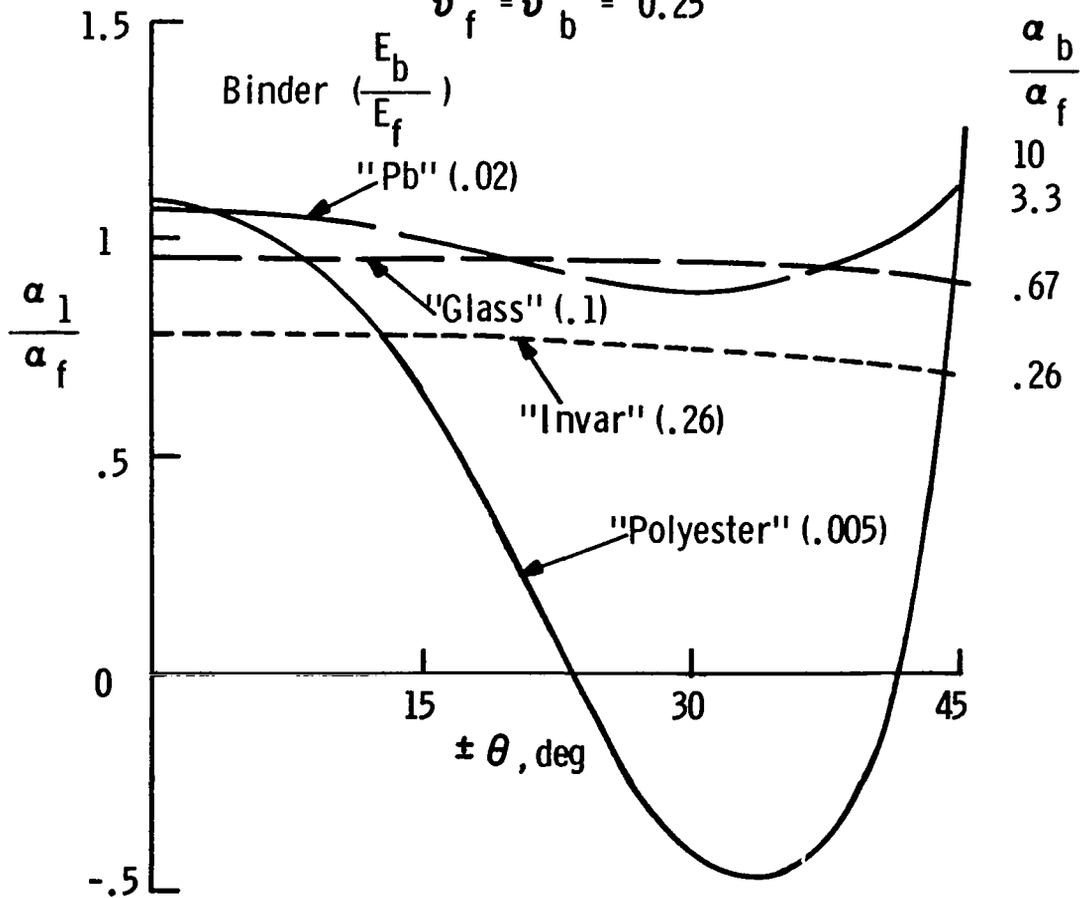


Figure 6. Ratio of coefficient of thermal expansion in the 1-direction to coefficient of thermal expansion of the filaments for two-dimensional reinforcement at angles $\pm \theta$ to the 1-direction. Case III: Binders of constant values of the product of Young's modulus and coefficient of thermal expansion.

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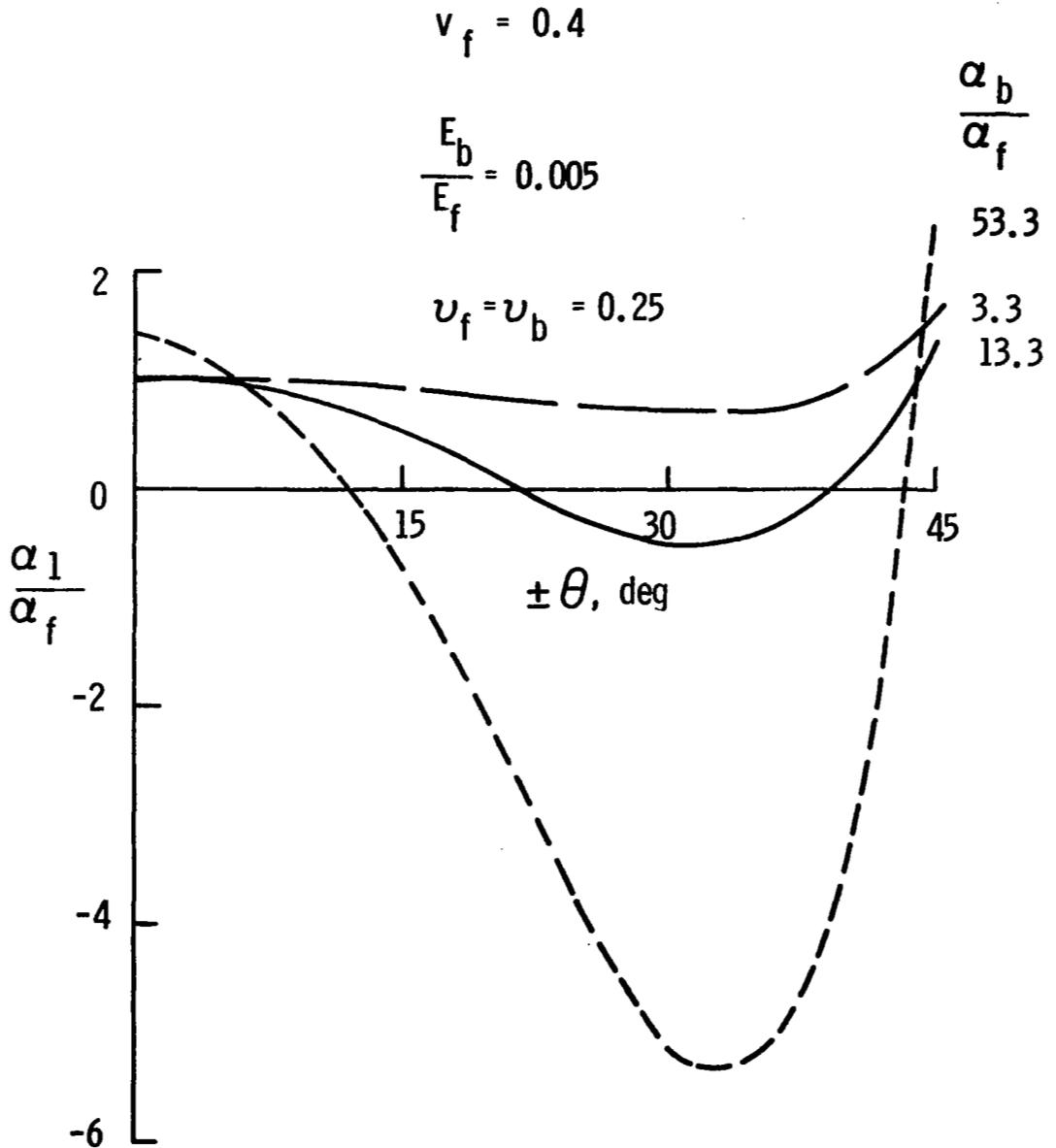


Figure 7. Effect of variation in binder coefficient of thermal expansion on the ratio of coefficient of thermal expansion in the 1-direction to coefficient of thermal expansion of the filaments for two-dimensional reinforcement at angles $\pm \theta$ to the 1-direction.

" TUNGSTEN CARBIDE " FILAMENTS

$$\nu_f = 0.4$$

$$\frac{E_b}{E_f} = 0.005$$

$$\frac{\alpha_b}{\alpha_f} = 13.3$$

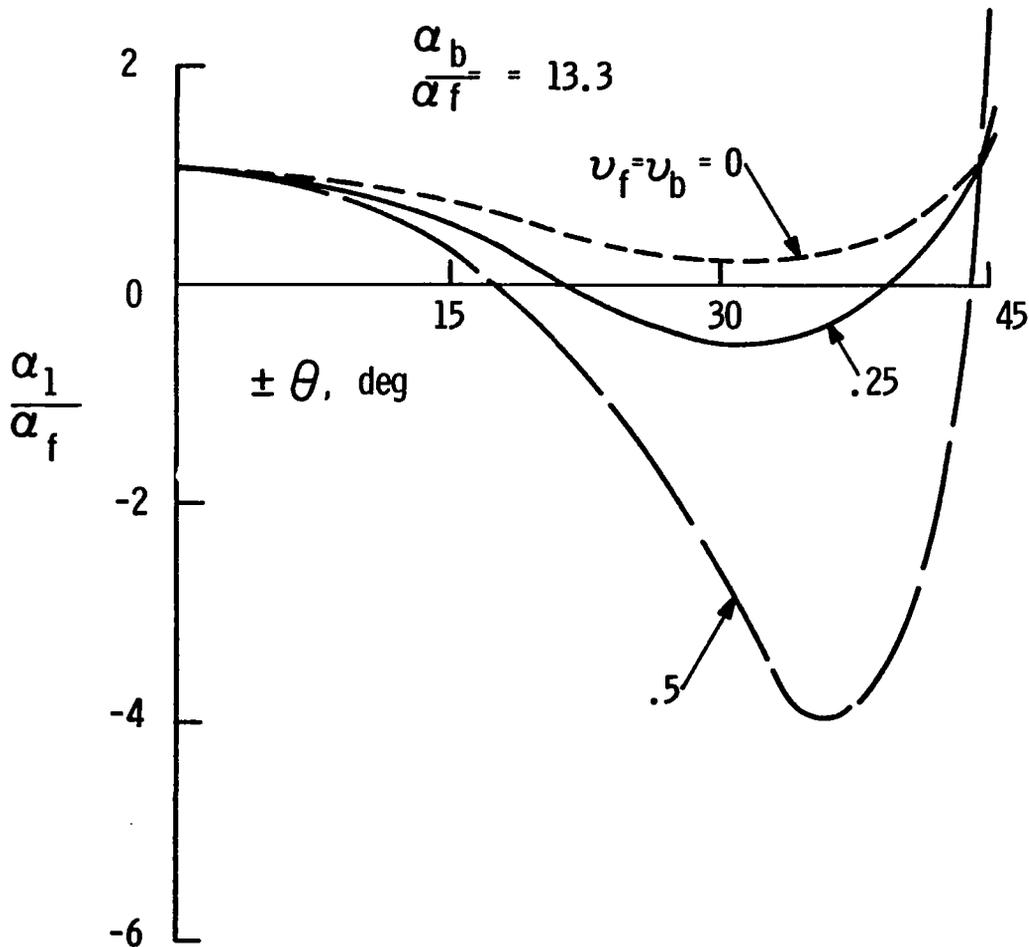


Figure 8. Effect of variation in Poisson's ratio on the ratio of coefficient of thermal expansion in the 1-direction to coefficient of thermal expansion of the filaments, for two-dimensional reinforcement at angles $\pm \theta$ to the 1-direction.

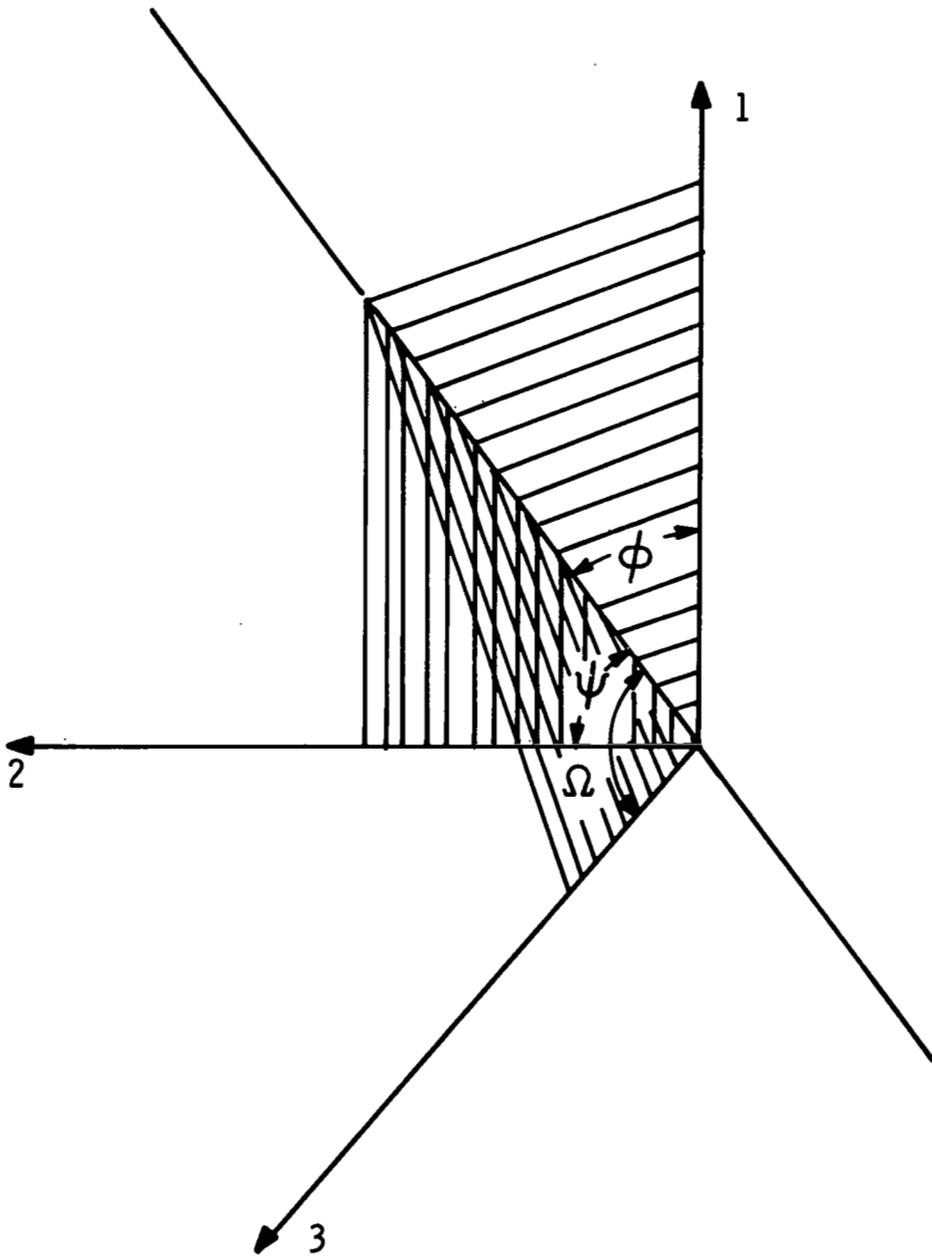


Figure 9. Designation of angles Φ , ψ , and Ω used in analysis of three-dimensional reinforcement.

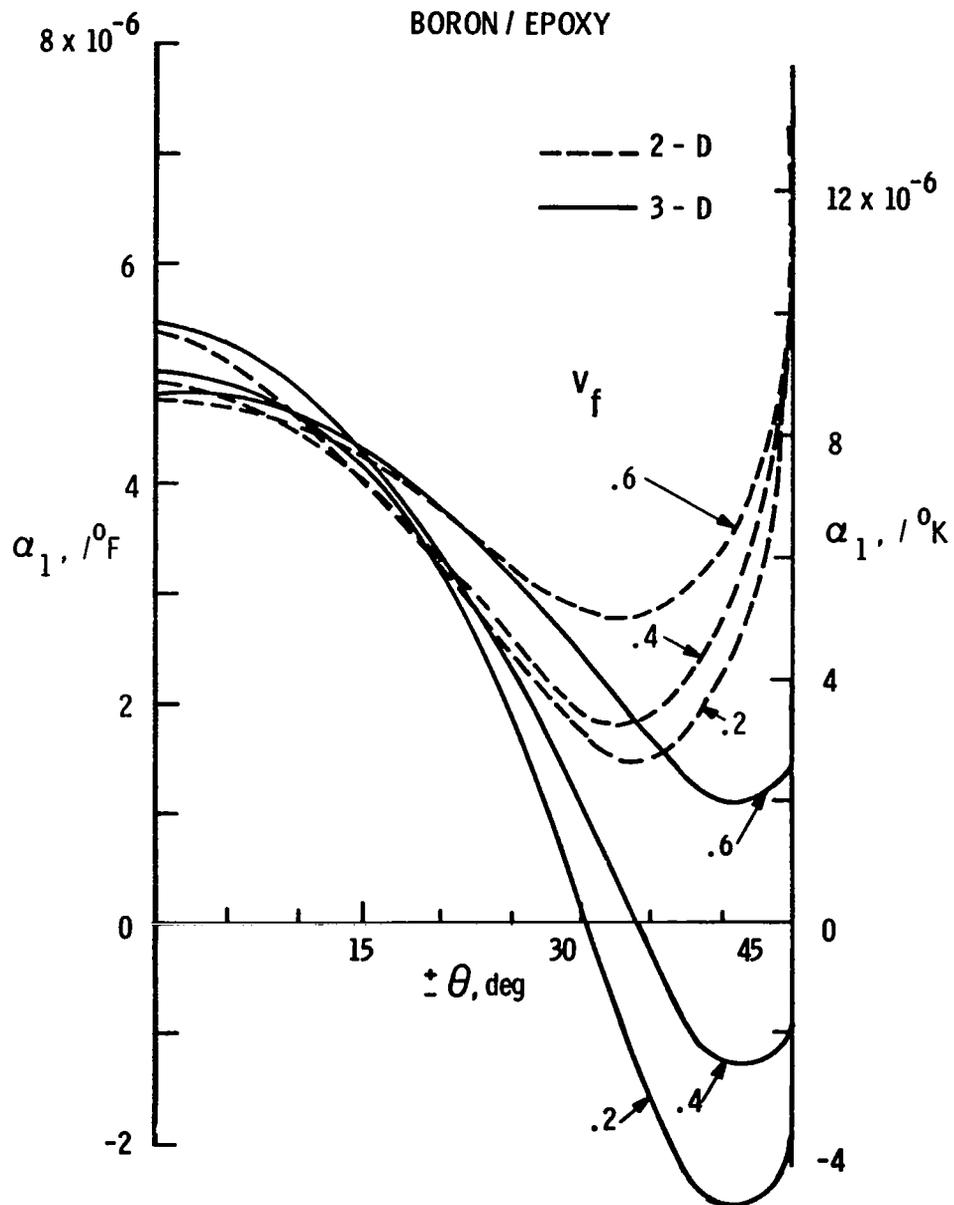


Figure 10. Coefficients of thermal expansion in the 1-direction for various two- and three-dimensional reinforcement configurations of boron/epoxy.

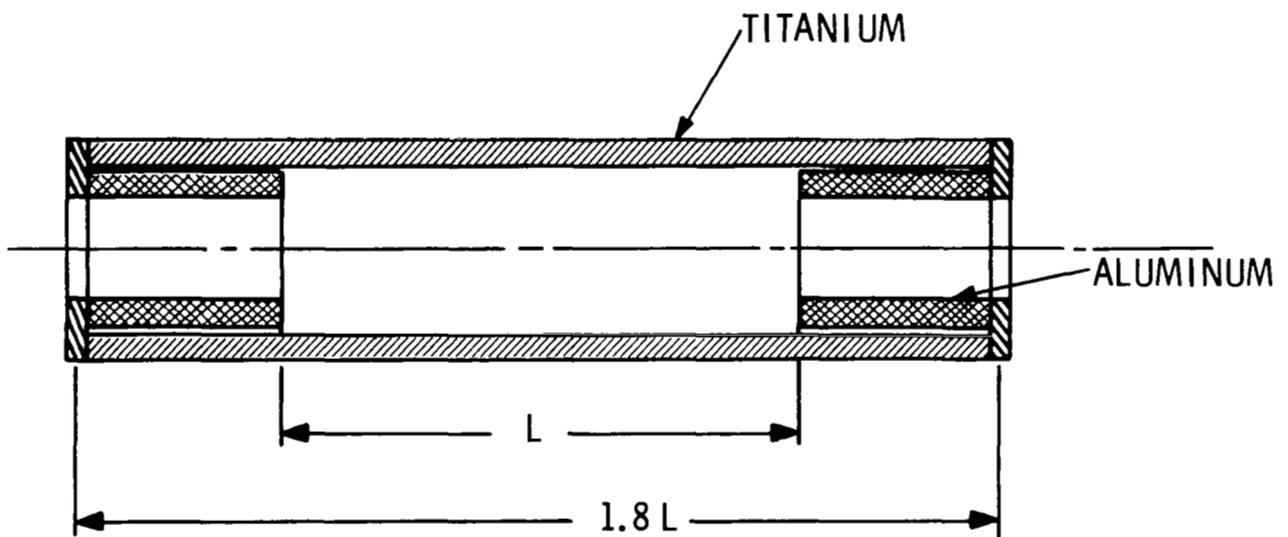


Figure 11. Schematic, cross-sectional representation of bi-metallic element compensated to maintain a constant length L with variations in temperature.

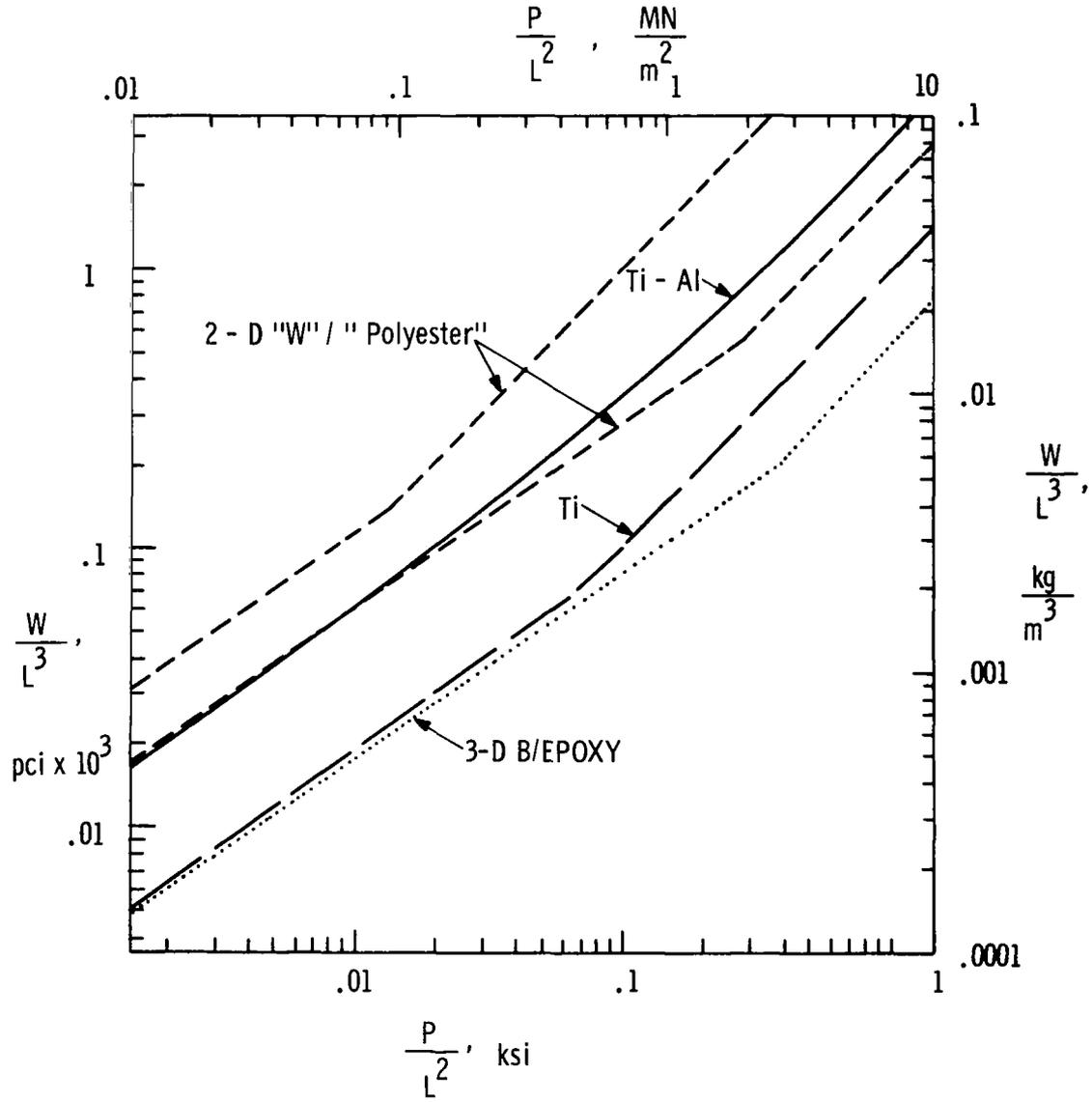


Figure 12. Structural weight/efficiency plot for tube-columns of various constructions.

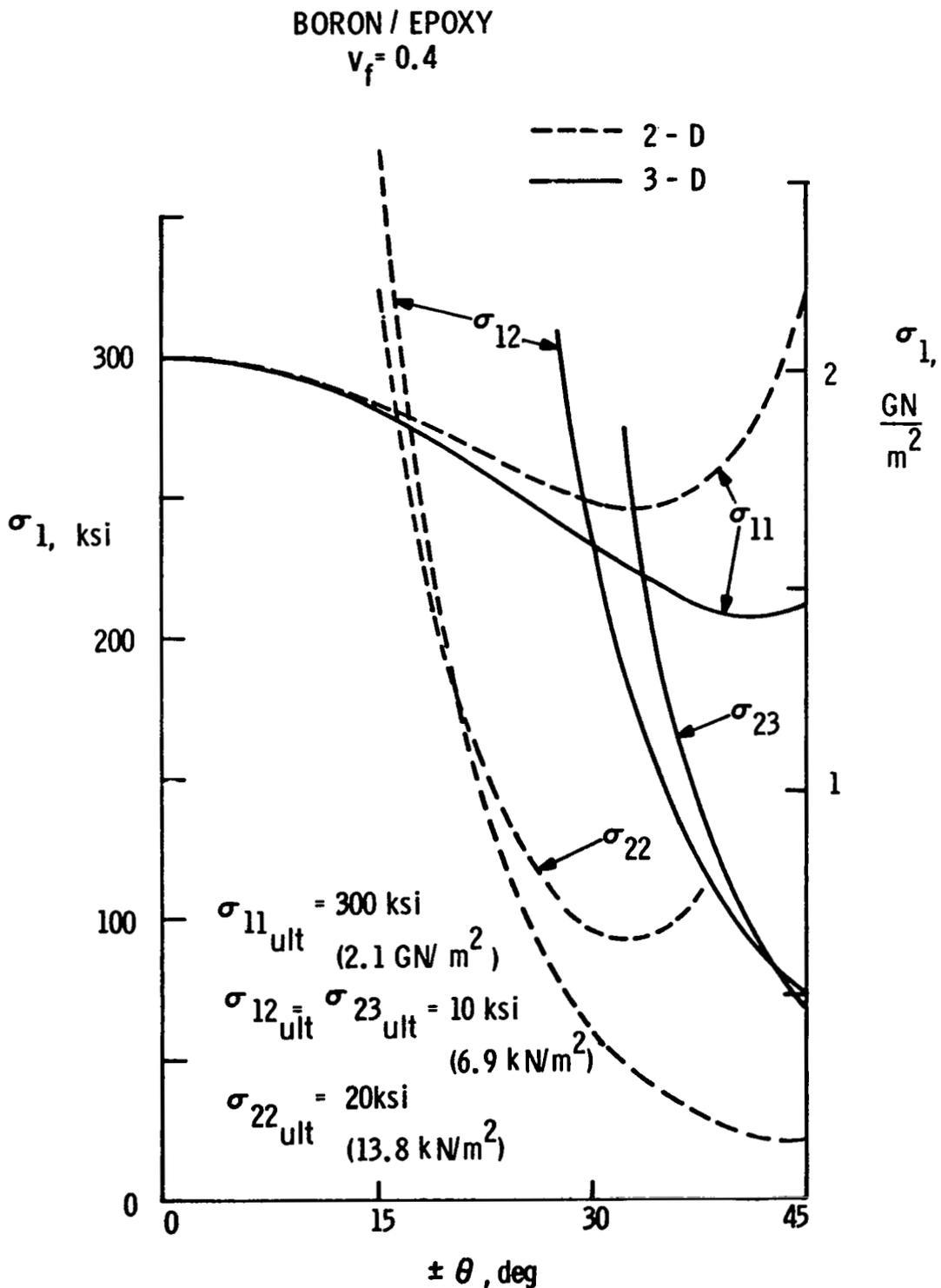


Figure 13. Results of analyses of strengths of two- and three-dimensional reinforcement configurations of boron/epoxy.